PHYS 4247 — Assignment #1

Due: 9/14/17

1. (a) Suppose you are in an infinitely large, infinitely old universe in which the average number density of galaxies is $n_* = 1$ Mpc^{-3} and the average galaxy radius is $R_g = 2000$ pc. How far, on average, could you see in any direction before your line of sight struck a galaxy? (Assume standard Euclidean geometry holds true.)

   (b) Now, suppose you are in an expanding universe full of regulation baseballs each of mass $m_{\rm bb} = 0.145$ kg and radius $r_{\rm bb} = 0.0369$ m. If the balls are distributed uniformly throughout the universe, what would be the current observed number density of baseballs (in Mpc^{-3}) for a $k = 0$ universe? Given this density, how far would you be able to see, on average, before your line of sight intersected a baseball? (Assume standard Euclidean geometry, non-relativistic baseballs, and $H_0 = 70$ km s^{-1} Mpc^{-1}.)

2. A hypothesis once used to explain the Hubble relation is the "tired light hypothesis". The tired light hypothesis states that the universe is not expanding, but that photons simply lose energy as they move through space (by some unexplained means) with the energy loss per unit distance being given by the law

$$\frac{dE}{dr} = -KE, \tag{1}$$

   where K is a constant. Show that this hypothesis gives a distance-redshift relation that is linear in the limit $z \ll 1$. What must the value of K be in order to yield a Hubble constant of $H_0 = 70$ km s^{-1} Mpc^{-1}?

3. Consider a more general equation of state, $p = (\gamma - 1)\rho c^2$, where γ is a constant in the range $0 < \gamma < 2$. Find solutions for $\rho(a)$, $a(t)$ and hence $\rho(t)$ for Universes containing such matter. Assume $k = 0$ in the Friedmann equation.

4. Using your answer to Problem #3, what value of γ would be needed so that ρ has the same time dependence as the term kc^2/a^2? Find the solution $a(t)$ to the full Friedmann equation for a fluid with this γ, assuming negative k.

5. Now consider the case $k < 0$, with a Universe containing only matter. What is the solution $a(t)$ in a situation where the final term of the Friedmann equation dominates over the density term? How does the density of matter vary with time? Is domination by this final term a stable situation that will continue forever?