PHYS 4247 — Assignment #3

Due: 10/19/17

1. (a) Demonstrate that for spatially-flat matter-dominated cosmologies with a cosmological constant the Friedmann equation can be written as

$$H^2(z) = H_0^2(1 - \Omega_{m,0} + \Omega_{m,0}(1+z)^3). \tag{1}$$

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(b) Use this to show that for spatially-flat cosmologies the coordinate distance to redshift z is

$$r_0 = cH_0^{-1} \int_0^z \frac{dz}{(1 - \Omega_{m,0} + \Omega_{m,0}(1+z)^3)^{1/2}}. \tag{2}$$

(c) Bearing in mind that $cH_0^{-1} = 3000h^{-1}$ Mpc, derive formulae for the luminosity and angular diameter distances as a function of z for the special case $\Omega_{m,0} = 1$.

2. (a) You observe a quasar at a redshift of $z = 5$ and determine that the observed flux varies on a timescale of $\delta t_0 = 3$ days. If the observed variation in flux is due to a variation in the intrinsic luminosity of the quasar, what was the variation timescale δt_e at the time the light was emitted?

(b) For the light from the quasar to vary on a timescale δt_e, the bulk of the light must come from a region with a physical size $R \leq R_{\rm max} = c(\delta t_e)$. What is $R_{\rm max}$ for the observed quasar in pc?

(c) What is the angular size of $R_{\rm max}$ for a matter-dominated universe with no cosmological constant, $\Omega_{m,0} = 0.3$ and $H_0 = 70$ km s^{-1} Mpc^{-1}?

3. Show that in a $k = 0$, $\Lambda = 0$ universe, the following relation between density and time holds:

$$\rho = \frac{1}{6\pi G t^2} \tag{3}$$

4. Show that the present proper distance of the farthest galaxy we can see now is

$$d_{\rm prop} = ca_0 \int_{t_{\rm min}}^{t_0} \frac{dt}{a(t)} \tag{4}$$

5. The Draco galaxy is a dwarf galaxy in the Local Group and has a luminosity of $L = 1.8 \times 10^5\ L_\odot$. Half of this luminosity is contained within a sphere of radius $r_h = 120$ pc. The measured velocity dispersion of the galaxy's stars is $\sigma = 10.5$ km s^{-1}. What is the mass and mass-to-light ratio of the Draco galaxy (if you make any assumptions, state them)? Compare the mass-to-light ratio to the Galactic value ($\approx 40 M_\odot/L_\odot$) and to the Coma cluster ($\approx 250 M_\odot/L_\odot$) and comment on the relative amount of dark matter in the Draco galaxy. (Note: for isotropic orbits, $\langle v^2 \rangle = 3\sigma^2$)