PHYS 4247 — Assignment #4
Due: 11/2/17

1. Assuming a typical galaxy has a baryonic mass of 10^{11} $M_\odot$ estimate the number of galaxies and the number of protons in the observable Universe. (For this problem use $\Omega_{m,0} = 0.3$, $\Omega_b = 0.046$, $H_0 = 70$ km s^{-1} Mpc^{-1}, $\rho_{\rm crit} = 2.78h^{-1} \times 10^{11}$ $M_\odot/(h^{-1}$ Mpc$)^3$, and a flat, matter dominated Universe).

2. (a) Throughout this question, assume that the Universe only contains matter and a possible cosmological constant. Use the definition of the density parameter to show that at any epoch we can write the matter density as

$$\frac{H^2\Omega_m(z)}{(1+z)^3} = \text{constant} = \Omega_{m,0}H_0^2. \tag{1}$$

(b) Given that in any realistic cosmology, $\Omega_m \approx 1$ at early times, compute the Hubble parameter at decoupling as a fraction of its present value. As the only important characteristic scale in the young Universe, the Hubble length cH^{-1} gives the characteristic scale of the first peak of the CMB.

(c) Now show that in a spatially-flat cosmology with a cosmological constant, the present co-moving distance to an object with $z \gg 1$ is given approximately by

$$r_0 \approx \frac{2cH_0^{-1}}{\sqrt{\Omega_{m,0}}}. \tag{2}$$

(d) By considering the angular diameter distance, demonstrate the the angle subtended by the Hubble length at decoupling is approximately independent of $\Omega_{m,0}$ in spatially-flat cosmologies, and compute its value in degrees. This demonstrates that the peak position is nearly independent of Ω_Λ for spatially-flat geometries, the approximations being the formula for r_0 above and the assumption that the Universe is perfectly matter dominated at last-scattering.

3. (a) Use the Mattig formula to show that in a matter-dominated open Universe with $\Lambda = 0$, the angular diameter distance to an object at redshift z is

$$d_{\rm ang} = 2cH_0^{-1}\frac{\Omega_{m,0}z + (\Omega_{m,0} - 2)(\sqrt{1+\Omega_{m,0}z} - 1)}{\Omega_{m,0}^2(1+z)^2}. \tag{3}$$

(b) Use this formula and the result of #2(b) to demonstrate that the angular size of the Hubble length at decoupling is approximately $\theta = 1$ deg$\times\Omega_{m,0}^{1/2}$ (Hint: consider how the terms behave at $z \gg 1$). Given that if $\Omega_{m,0} = 1$ the peak in the microwave power spectrum is at $\ell \approx 220$, use this result to predict the peak position in an open Universe

with $\Omega_{m,0} = 0.3$ and compare with the observed power spectrum shown in class. This demonstrates that the peak position does depend significantly on geometry, and hence can strongly constrain it.

4. The total luminosity of the stars in our galaxy is $L \approx 2.3 \times 10^{10}$ $L_\odot$. Suppose that the luminosity of our galaxy has been constant for the past 10 Gyr. How much energy has our galaxy emitted in the form of starlight during that time? Most stars are powered by the fusion of H into ^{4}He, with the release of 28.4 MeV for every helium nucleus formed. How many helium nuclei have been created within stars in our galaxy over the course of the past 10 Gyr, assuming that the fusion of H into ^{4}He is the only significant energy source? If the baryonic mass of our galaxy is 10^{11} $M_\odot$, by what amount has the helium fraction $Y \equiv \rho(^4\mathrm{He})/\rho_b$ of our galaxy been increased over its primordial value $Y_4 = 0.24$?

5. The typical energy of a neutrino in the Cosmic Neutrino Background is $\sim 5 \times 10^{-4}$ eV. Given that the typical cross-section for the interaction of a neutrino with any other particle (via the weak nuclear force) is $\sigma_w \sim 10^{-47}(kT/1\ \mathrm{MeV})^2\ \mathrm{m}^2$, what is the approximate cross-section for one of these cosmic neutrinos? Suppose you had a large lump of ^{56}Fe with density $\rho = 7900$ kg m^{-3}. How far, on average, would a cosmic neutrino travel through iron before interacting with a proton, neutron or electron? (Assume that the cross-section for interaction is simply σ_w regardless of the type of particle that the neutrino interacts with.)