PHYS 4247 — Assignment #5

Due: 11/28/17

1. (a) In class we learned that in a radiation dominated Universe, the temperature and time are related by
$$\left(\frac{1\ \mathrm{sec}}{t}\right)^{1/2} \approx \frac{T}{2\times 10^{10}\ \mathrm{K}}. \tag{1}$$
   At what time was the temperature 3×10^{25} K?

   (b) Suppose that at that time Ω_{total} were a bit less than one, so that the Universe quickly became curvature dominated, with expansion law $a(t) \propto t$. How would the above equation be changed, and how old would the Universe have been when its temperature fell to 3 K?

2. (a) Magnetic monopoles behave as non-relativistic matter. Suppose that at a temperature corresponding to the Grand Unified era, about 3×10^{28} K, magnetic monopoles were created with a density of $\Omega_{\mathrm{mon}} = 10^{-10}$. Assuming that the Universe has a critical density and is radiation dominated, what was the temperature when the density of monopoles equalled that of the radiation?

   (b) In the present Universe, $T \approx 3$ K. Compute the value $\Omega_{\mathrm{mon}}/\Omega_{\mathrm{rad}}$ would have at the present day. Is this ratio compatable with observations?

3. Consider the situation of Problem #2. If we have a period of inflation, the monopole density still reduces as $\rho_{\mathrm{mon}} \propto 1/a^3$, but the total density, dominated by the cosmological constant, remains fixed. Since that density will be converted to radiation after inflation, we can imagine that the radiation density remains constant during inflation. How much inflationary expansion is necessary so that the present density of monopoles matches that of radiation?

4. Consider an empty, negatively curved, expanding universe. If a dynamically insignificant amount of matter ($\Omega_m \ll 1$) is present in such a universe, how do density fluctuations in the matter evolve with time? That is, what is the functional form of $\delta(t)$?

5. A volume containing a photon-baryon fluid is adiabatically expanded or compressed. The energy density of the fluid is $\epsilon = \epsilon_\gamma + \epsilon_{\mathrm{bary}}$, and the pressure is $P = P_\gamma = \epsilon_\gamma/3$. What is $dP/d\epsilon$ for the photon-baryon fluid? What is the sound speed, c_s? Estimate how large the baryon contribution is to the Jean's length, λ_J, at the time just *before* decoupling?

   (Hint: Start by calculating how $\Omega_{\mathrm{rad}}/\Omega_{\mathrm{baryon}}$ evolves with the scale factor. Evaluate this using the values for the current time (i.e., now) and the z at decoupling ($z = 1100$). That will give you how much energy density is in the baryons at decoupling relative to the photons. From there you can write down an expression for ϵ just in terms of ϵ_γ. Go on to show how big a correction the baryons make to a calculation of the Jeans length that only assumed radiation in the energy density.)