but $\text{LHS} = \int^{r_0} \frac{dr}{\sqrt{1-Kr^2}} = \sin^{-1} r_0$ for $K=+1$

$$\therefore \sin^{-1} r_0 = \theta_0 - \theta_1$$

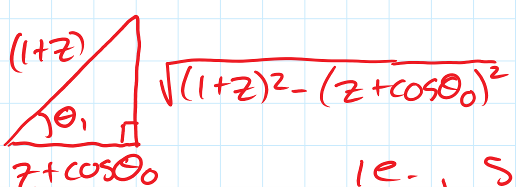
or $\quad r_0 = \sin(\theta_0 - \theta_1) = \sin\theta_0 \cos\theta_1 - \cos\theta_0 \sin\theta_1$

must express θ's in terms of z

$$(1+z) = \frac{a(t_0)}{a(t_1)} = \frac{\sin^2(\theta_0/2)}{\sin^2(\theta_1/2)}$$

but $\sin^2(\theta_1/2) = \frac{1}{2} - \frac{1}{2}\cos\theta_1$

$$\cos\theta_1 = \frac{1 - 2\sin^2(\theta_0/2)}{1+z} = \frac{z + \cos\theta_0}{1+z}$$



ie., $\sin\theta_1 = \frac{\sqrt{(1+z)^2 - (z+\cos\theta_0)^2}}{(1+z)}$

$$\sin^2\left(\frac{\theta_0}{2}\right) = \frac{a_0}{a_m} = \frac{\Omega_{m,0} - 1}{\Omega_{m,0}}$$

but $\cos\left(\frac{\theta_0}{2}\right) = \left(1 - \sin^2\left(\frac{\theta_0}{2}\right)\right)^{1/2} = \left(\frac{1}{\Omega_{m,0}}\right)^{1/2}$

sub. into $r_0 = \cdots$

$$r_0 = \frac{4(\Omega_{m,0} - 1)^{1/2}}{\Omega_{m,0}^2 (1+z)} \left[\frac{\Omega_{m,0} z}{2} + \left(1 - \Omega_{m,0}/2\right)\left(1 + \Omega_{m,0} z\right)^{1/2} \right]$$

$$\Omega_{m,0}^2(1+z)\Big[\ \ 2 \qquad\qquad 1/2)(\qquad\qquad \Big]$$

'Mattig formula' $K=+1$

For $K=-1$, use $a = \frac{a_m}{2}(\cosh\Psi - 1)$ where

$$a_m = \frac{\Omega_{m,0}}{1-\Omega_{m,0}} a_0$$

Get the same thing except $(1-\Omega_{m,0})^{1/2}$ out front

From $\frac{Kc^2}{H_0^2 a_0^2} = (\Omega_{m,0} - 1)$, $\quad (\Omega_{m,0}-1)^{1/2} = \frac{c}{H_0 a_0} \;/ K=1$

$$(1-\Omega_{m,0})^{1/2} = \frac{c}{H_0 a_0} \;/ K=-1$$

$$\therefore r_0 = \frac{4c}{H_0 a_0}\frac{1}{\Omega_{m,0}^2}\frac{1}{(1+z)}\left[\frac{\Omega_{m,0} z}{2} + \left(1-\frac{\Omega_{m,0}}{2}\right)\left(1-\left(1+\Omega_{m,0} z\right)^{1/2}\right)\right]$$

valid for both $K=+1$ & -1

in terms of $q_0 = \frac{\Omega_{m,0}}{2}$

$$r_0 = \frac{c}{H_0 a_0 q_0^2 (1+z)}\left[q_0 z + (1-q_0)\left(1-(1+2q_0 z)^{1/2}\right)\right]$$

Now, $f = \frac{L}{4\pi d_l^2}$

where $d_l = r_0 a(t_0)(1+z) = \frac{c}{H_0}\frac{1}{q_0^2}\left(q_0 z + (1-q_0)\left(1-(1+2q_0 z)^{1/2}\right)\right)$

In optical/IR astronomy use magnitudes for flux, where $m = -2.5 \log_{10} f + \text{const.}$

where const. is set by defining absolute mag. at a certain distance (M). Then $m - M = 5 \log_{10} d_L - 5$ where d_L is measured in units of pc.

magnitude-redshift tests

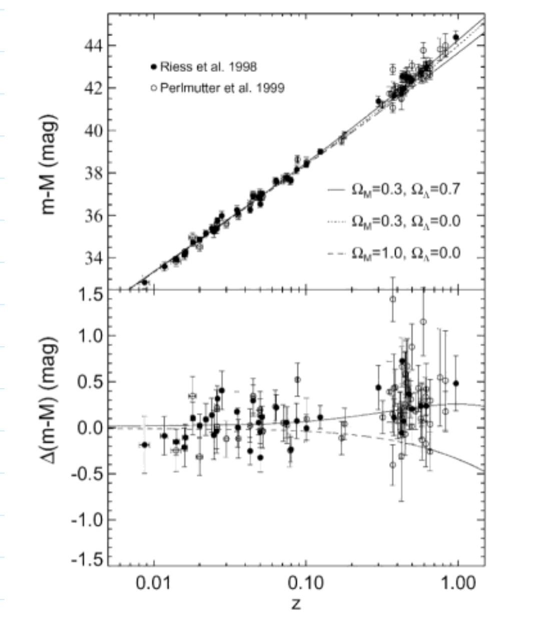


Figure 7.5: Distance modulus versus redshift for type Ia supernovae from the Supernova Cosmology Project (Perlmutter et al.

- issues w/ m-z test

1) need a standard candle = an object in which the intrinsic L is known (e.g. Cepheid variable, Type 1a SN)

2) K-correction

3) evolutionary corrections (metallicity, dust, gas fractions at diff. z)

K-Corrections

Let L = total energy/s emitted by the source

f = total energy/area/s received by the observer

In practice make observations in some narrow band of frequencies. Let $L(\nu_e)\,d\nu_e$ be the energy emitted by the source in $\nu_e \to \nu_e + d\nu_e$

And $f(\nu_0)\,d\nu_0$ where $\nu_e = \nu_0(1+z)$

$$f(\nu_0)\,d\nu_0 = \frac{L(\nu_e)\,d\nu_e}{4\pi d_e^2}$$

and $$\frac{d\nu_e}{d\nu_0} = (1+z)$$

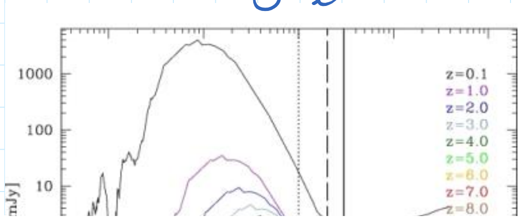
then $$f(\nu_0) = \frac{L\big(\nu_0(1+z)\big)(1+z)}{4\pi d_e^2}$$

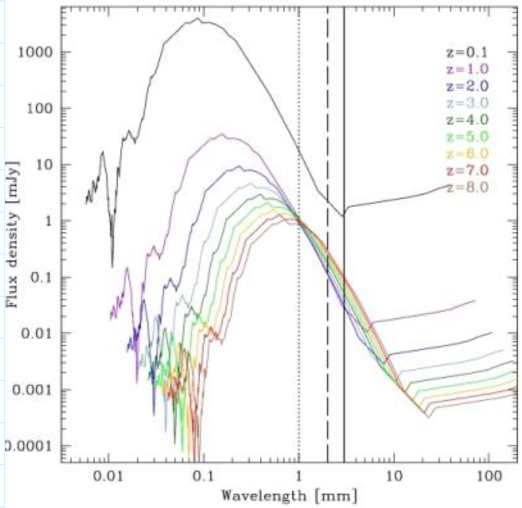
$$= \frac{L(\nu_0)}{4\pi d_e^2} \cdot \frac{L\big(\nu_0(1+z)\big)(1+z)}{L(\nu_0)}$$

define $$K(z) = -2.5 \log\left[\frac{(1+z)\,L\big(\nu_0(1+z)\big)}{L(\nu_0)}\right]$$

$$= m_{obs} - m_{true}$$

$$\therefore\ m_{obs} - M = 5\log d_e - 5 + K(z)$$





Need spectral model of source to correct (physics or a template)

recall $\frac{d_{ang}}{d_{lum}} = \frac{1}{(1+z)^2}$

hence, $d_{ang} = \frac{c}{H_0 \, q_0^2 (1+z)^2}\left[q_0 z + (1-q_0)\left(1-\sqrt{1+2q_0 z}\right)\right]$

where $\Theta = \frac{D}{d_A}$ where Θ is measured, but v.

hard to find a standard ruler