Classical Source Counts

Element of Volume $dV = 4\pi r^2 dr$

Let density of objects w/ luminosity L be n_0

$$dN = n_0 4\pi r^2 dr, \quad N = \int_0^r 4\pi r^2 n_0 dr = \frac{4\pi}{3} r^3 n_0$$

For a limiting flux f

$$f = \frac{L}{4\pi r^2}, \quad \text{for fixed } f, \quad r = \frac{L^{1/2}}{(4\pi f)^{1/2}}$$

$$\therefore N(>f) = \frac{4}{3}\pi n_0 \frac{L^{3/2}}{(4\pi f)^{3/2}}$$

i.e. $\log(N(>f)) = -\frac{3}{2}\log f + \text{const.}$

in cosmological case,

$$dV = \frac{a^3 r^2 \sin\theta \, d\theta \, d\phi \, dr}{\sqrt{1-Kr^2}} = \frac{4\pi a^3 r^2 dr}{\sqrt{1-Kr^2}}$$

Want to express this dV in terms of z

$$\frac{c\,dt}{a(t)} = \frac{dr}{\sqrt{1-Kr^2}} \rightarrow \frac{c}{\dot{a}}\frac{da}{a} = \frac{dr}{\sqrt{1-Kr^2}}$$

but $(1+z) = \dfrac{a_0}{a} \quad \therefore \dfrac{dz}{1+z} = \dfrac{da}{a}$

[illegible]

$$a \qquad 1+z \qquad a$$

from F.E. (no Ω_Λ), $\dot{a}^2 = \frac{8\pi G \rho a^2}{3} - Kc^2$

$$\dot{a}^2 = \frac{8\pi G \rho a^3}{3a} - Kc^2 = \frac{8\pi G \rho_0 a_0^3}{3a}\frac{H_0^2}{H_0^2} - Kc^2$$

$$= \frac{\rho_0 a_0^3 H_0^2}{\rho_{c,0}\, a} - Kc^2 = \Omega_{m,0}\frac{a_0^3 H_0^2}{a} - Kc^2$$

$$\left(\text{but } Kc^2 = H_0^2 a_0^2 (\Omega_{m,0} - 1)\right)$$

$$\dot{a}^2 = \Omega_{m,0}(1+z)a_0^2 H_0^2 - a_0^2 H_0^2 \Omega_{m,0} + a_0^2 H_0^2$$

$$\dot{a}^2 = a_0^2 H_0^2 (1 + \Omega_{m,0} z)$$

$$\text{so } \frac{dr}{\sqrt{1-Kr^2}} = \frac{c\,dz}{(1+z) H_0\, a_0 (1+\Omega_{m,0} z)^{1/2}}$$

$$\therefore dV = 4\pi \left(\frac{a}{a_0}\right)^3 a_0^3 \frac{r^2\, dr}{\sqrt{1-Kr^2}}$$

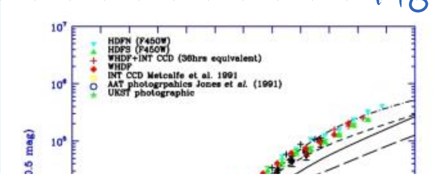
$$= \frac{4\pi}{(1+z)^3} \frac{a_0 a_0^2 r^2 (1+z)^2}{(1+z)^2} \frac{c\,dz}{(1+z) H_0 a_0 (1+\Omega_{m,0} z)^{1/2}}$$

$$= \frac{4\pi c}{H_0} \frac{d_L^2}{(1+z)^6} \frac{dz}{(1+\Omega_{m,0} z)^{1/2}}$$

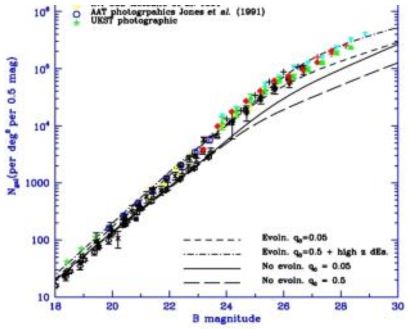
$$\text{so } dN(a) = n(a)\, dV(z(a)) = n_0 (1+z)^3 dV(z)$$

since $n(r)\, a^3 = n(a_0)\, a_0^3$

$$dN(z) = 4\pi n_0 \frac{c}{H_0} \frac{d_L^2}{(1+z)^3} \frac{dz}{(1+\Omega_{m,0} z)^{1/2}}$$



– in practice, evol. of



– in practice, evol. of source properties dominates counts

w/ magnitudes, use a limiting m, where $m - M = 5\log r - 5$
or $r = 10 \cdot 10^{0.2(m-M)}$

$$N(<m) = \int dN(z)$$

Measuring the Density Parameter, Ω_m

Need to measure the total mass density of the constituents of the Univ. & compare to $\rho_{c,0}$.

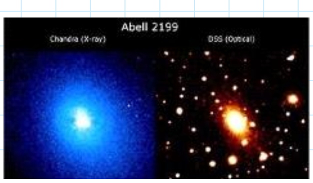
A) Stars

- add up all the light in a large region (several galaxies)
- stellar evol. theory gives good estimates of how massive stars w/ diff. lumin. are

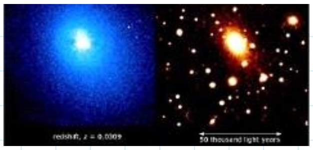
$$\Omega_{stars} \equiv \frac{\rho_{stars}}{\rho_c} \simeq 0.005 - 0.01$$

B) total baryonic mass

- stars are just a fraction of the total baryonic mass
- most of the baryons are in diffuse gas b/w the galaxies



The X-rays emit optically-thin



The X-rays emit optically-thin free-free rad'n, so measuring the $kT(r)$, $n(r)$ and assume hydrostatic balance. Find gas masses $\sim 10\times$ the stellar masses. $\Omega_{bary} \sim 0.1$

C) Nucleosynthesis

- the best current limits on the baryon density comes from prediction of primordial nucleosynthesis
- the efficiency w/ which fusion takes place in the early Univ. ($H \to D$, He, Li, ...) depends on the density of baryons
- measure the amount of D & Li in ~primordial clouds constrains $0.016 \leq \Omega_{bary} h^2 \leq 0.024$

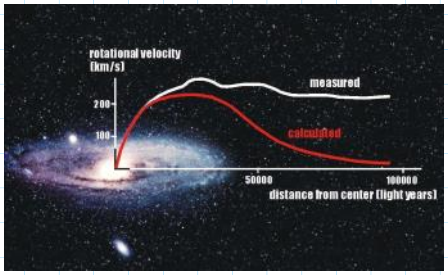
D) Non-baryonic matter or dark matter

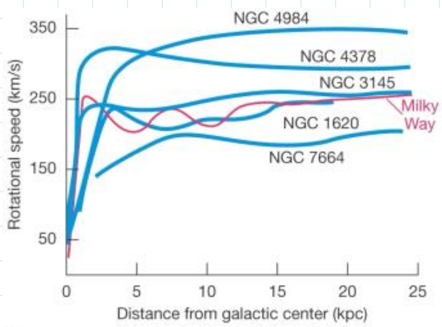
i) galaxy rotation curves

- stars in a spiral galaxy are on orbits given by Kepler's Law: $\frac{V^2}{R} = \frac{GM(R)}{R^2}$

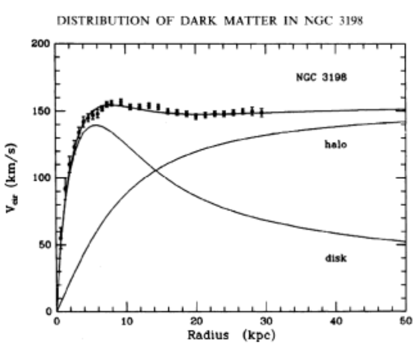
or $V = \sqrt{\frac{GM(R)}{R}}$

$\therefore$ at large distances, the rotational velocity $\propto R^{-1/2}$


rotational velocity
(km/s)
200
100
measured
calculated
50000
100000
distance from center (light years)


NGC 4984
NGC 4378
NGC 3145
Milky Way
NGC 1620
NGC 7664
350
250
150
50
Rotational speed (km/s)
0
5
10
15
20
25
Distance from galactic center (kpc)

(b)


DISTRIBUTION OF DARK MATTER IN NGC 3198
NGC 3198
halo
disk
V_{cir} (km/s)
Radius (kpc)
200
150
100
50
0
0
10
20
30
40
50