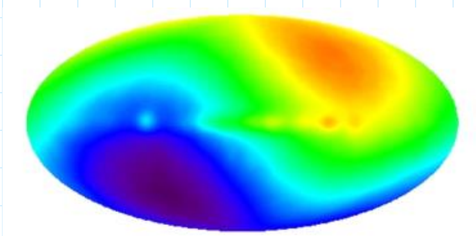
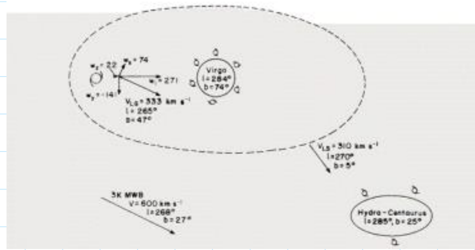
Properties of CMB Spectrum

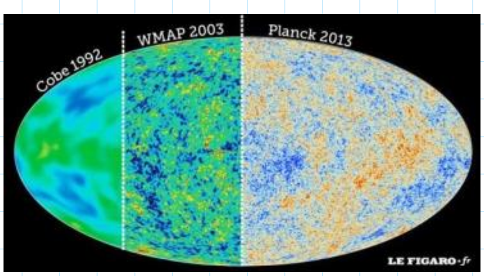
1) The spectrum at all (θ, ϕ) on the sky is an ideal BB to smaller than a fract. of a percent.

2) A temp. dipole of $\Delta T \sim 3\times10^{-3}\,K$ due to motion of the LG towards the Virgo cluster (which is itself being accelerated toward the Hydra-Centarus Supercluster)

$\therefore V_{LG} = 630 \pm 20 \ \frac{Km}{s}$







3) Subtract the dipole & foregrounds (ie, the galaxy) find very small fluctuations in temperature.

The mean temperature is

$$\langle T \rangle = \frac{1}{4\pi}\int T(\theta,\phi)\sin\theta\, d\theta\, d\phi = 2.725\,K$$

$$\frac{\delta T(\theta,\phi)}{T} = \frac{T(\theta,\phi) - \langle T \rangle}{\langle T \rangle}.$$ The rms fluctuation

is $\left\langle \left(\frac{\delta T}{T}\right)^2 \right\rangle^{1/2} = 1.1\times10^{-5}$

or a variation of $\sim 30\,\mu K$

- a background of a nearly isotropic BB rad'n is a natural outcome if the Univ. was once hot, dense, opaque & nearly homogeneous.

Origin of CMB

The CMB is an expected relic of a hot Big Bang. When the Univ. was much younger, it was much smaller, and the radiation within it was much more energetic (ie, it had a larger T). This T was high enough to keep all baryons ionized. The Univ. was also denser, so the photons were coupled to the e^-/ion plasma. As the Univ. expanded, 3 things happened to release the photons as a background.

i) recombination: the time when the baryons go from ionized to neutral ($n_{ion} = n_{neutral}$)

ii) decoupling: the time when the rate of photons scattering off of e^- < Hubble parameter

iii) last scattering: the time at which a typical

CMB photon underwent its last scattering w/ an e^-

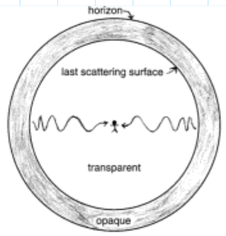


Figure 9.3: An observer is surrounded by a spherical last scattering surface. The photons of the CMB travel straight to us from the last scattering surface, being continuously redshifted.

Some ionization basics. Assume H Unive.

Define fractional ionization

$$X = \frac{n_p}{n_p + n_H} = \frac{n_p}{n_{bary}} = \frac{n_e}{n_{bary}}$$

$X = 1$, fully ioniz.

$X = 0$, neutral

The ionization energy of hydrogen is $Q = 13.6\,\text{eV}$

So, $h\nu > Q$ for ionization $H + \gamma \to p + e^-$

radiative recombination: $p + e^- \to H + \gamma$

Recall, $\eta \simeq 5\times10^{-10}$, if $\langle h\nu \rangle > Q$, then $X = 1$ will be reached very easily.

During the ionized epoch photons underwent Thomson scattering: $\gamma + e^- \to \gamma + e^-$ w/ $\sigma = 6.65\times10^{-29}\,\text{m}^2$. The mean-free-path of a photon (ie, the avg. dist. it can travel before scattering) is $\lambda = \frac{1}{n_e \sigma}$

The scattering rate $\Gamma = \frac{c}{\lambda} = n_e \sigma c$

When $X = 1$, $n_e = n_p = n_{bary}$. Now, $n_{baryon,0} = 0.22\,m^{-3}$

and $n_{bary} = \frac{n_{bary,0}}{a^3}$. So, if $X=1$, $\Gamma = \frac{n_{bary,0}\,\sigma c}{a^3}$

$= \frac{4.4 \times 10^{-21}\,s^{-1}}{a^3}$

The photons remain coupled to the e^- as long as $\Gamma > H$, or $\lambda < \frac{c}{H}$.

During this time of freq. interactions, the photons, e^- & p all have the same T. When decoupling occurs, the T also decouple.

Calculation of Recombination & Decoupling Epochs.

Consider the reaction in static. equil. when photons are coupled:

$$H + \gamma \rightleftharpoons p^+ + e^-$$

In stat. equil., n_x (the # density of particle w/ mass m_x) is

$$n_x = g_x \left(\frac{m_x kT}{2\pi\hbar^2} \right)^{3/2} e^{-m_x c^2/kT} \quad \text{if } kT \ll m_x c^2$$

g_x = stat. weight. = 2 for e^- & p^+ due to spin states

For, p^+, e^- & H

$$\frac{n_H}{n_p n_e} = \frac{g_H}{g_p g_e}\left(\frac{m_H}{m_p m_e}\right)^{3/2}\left(\frac{kT}{2\pi\hbar^2}\right)^{-3/2} e^{(m_p+m_e-m_H)c^2/kT}$$

After $m_p = m_H$, $(m_p+m_e-m_H)c^2 = Q$ & $g_H = 4$,

$$\frac{n_H}{n_p n_e} = \left(\frac{m_e kT}{2\pi\hbar^2}\right)^{-3/2} e^{Q/kT} \quad \text{(Saha equation)}$$

From def'n of X: $n_H = \frac{1-X}{X} n_p$ & $n_p = n_e$

$$\therefore \frac{1-X}{X} = n_p\left(\frac{m_e kT}{2\pi\hbar^2}\right)^{-3/2} e^{Q/kT}$$

For a H-only Univ. $\eta = \frac{n_p}{X n_\gamma}$

For a BB spect: $n_\gamma = \frac{2.404}{\pi^2}\left(\frac{kT}{\hbar c}\right)^3 = 0.243\left(\frac{kT}{\hbar c}\right)^3$

$$\therefore n_p = 0.243 X \eta \left(\frac{kT}{\hbar c}\right)^3$$

$$\therefore \frac{1-X}{X^2} = 3.84\,\eta\left(\frac{kT}{m_e c^2}\right)^{3/2} e^{Q/kT}$$

Quad. eqn. in X w/ +ve root $X = \frac{-1+\sqrt{1+4S}}{2S}$

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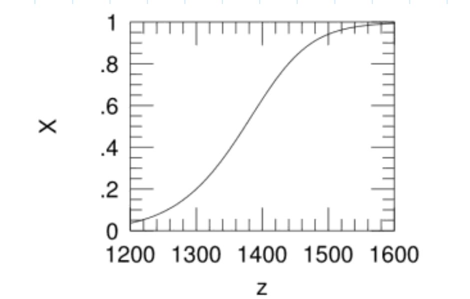
where $S(T,\eta) = 3.84\,\eta \left(\dfrac{kT}{mc^2}\right)^{3/2} e^{Q/kT}$

Define recombination when $X = 0.5$; use $\eta = 5.5\times10^{-10}$, then recombination temp. is

$$kT = 0.323 \text{ eV} \simeq \frac{Q}{42} \quad \text{or} \quad T = 3740\text{ K}$$

$\rightarrow z \simeq 1370$, $t \simeq 240{,}000$ yrs.



Photon decoupling occurs soon after recomb.

$$\Gamma(z) = n_e(z)\,\sigma c = X(z)(1+z)^3 n_{bary,0}\,\sigma c$$

$$= 4.4\times10^{-21} X(z)(1+z)^3$$

While recomb. is occuring, the Univ. is matter dominated, so F.E. is (w/ $k=0$)

$$\left(\frac{\dot a}{a}\right)^2 = \frac{8\pi G\rho}{3} = \frac{8\pi G}{3}\frac{\rho_0}{a^3} = \frac{8\pi G}{3H_0^2}\,\frac{H_0^2\rho_0}{a^3}$$

$$= \frac{\rho_0}{} \, H_0^2 \ldots \Omega_{m,} H_0^2$$

$$\rho_{c,0}\, a^3 \quad = \Omega_{m,0} \ldots$$

$$\frac{H^2}{H_0^2} = \frac{\Omega_{m,0}}{a^3} = \Omega_{m,0}(1+z)^3$$

The z of photon decoupling is found by setting $\Gamma = H$

$$4.4\times10^{-21}\, X(z_d)\,(1+z_d)^3 = \Omega_{m,0}^{1/2} H_0 (1+z_d)^{3/2}$$

For, $\Omega_{m,0} = 0.3$; $H_0 = 70$ and using the Saha-derived $X(z)$ gives $z_d \simeq 1130$

A more careful calc. that takes into account the non-equil. deriv. of X at small X gives

$z_d \simeq 1100$ $(T \simeq 3000\text{ K})$ $(t \simeq 350{,}000\text{ yrs})$

The CMB photons we detect come from the last scattering surface which may be slightly offset from z_d. We see photons from the $\tau = 1$ surface where $\displaystyle \tau = \int_a^{a_0=1} \Gamma(a)\frac{da}{\dot a} = \int_a^1 \frac{\Gamma(a)}{H(a)}\frac{da}{a}$

τ = optical depth. $\displaystyle \tau(z) = \int_0^z \frac{\Gamma(z)}{H(z)}\frac{dz}{(1+z)} = 1$

To a good approx. $z_{ls} \simeq z_d$.

So, after $z_d \simeq 1100$ the baryons were free to evolve under gravity and the photons continued to cool.