Inflation and the Very Early Universe

3 Classic Problems w/ standard B.B. Theory

1) Horizon Problem

Particle Horizon:

$$\int_{t_{min}}^{t_0} \frac{c\,dt}{a(t)} = \int_0^{r_h} \frac{dr}{\sqrt{1-Kr^2}}$$

If this integral converges as $t_{min} \to 0$ there is said to be a particle horizon w/ coordinate distance r_h.

For $K=0$, $a(t) \propto t^{2/3} = \alpha t^{2/3}$

$$\int_0^{t_0} \frac{c\,dt}{\alpha t^{2/3}} = r_h \qquad \therefore \quad r_h = \frac{3 c t_0^{1/3}}{\alpha}$$

Proper distance $= a(t_0)\, r_h$

$$d_h = a(t_0) \frac{3 c t_0^{1/3}}{\alpha} = \frac{\cancel{\alpha} t_0^{2/3}\, 3 c t_0^{1/3}}{\cancel{\alpha}} = 3 c t_0 = \frac{2c}{H_0}$$

$\to$ distance light could travel during the current age of the Univ.

Note: its > than ct_0 due to the expansion of the Univ.

The current proper distance to the last scattering surface $d_p(t_0) = c \int^{t_0} \frac{dt}{a(t)} \simeq 0.98\, d_h$

surface $d_p(t_0) = c\int_{t_{ls}}^{t_0} \frac{dt}{a(t)} \simeq 0.98\, d_h$

Thus, 2 opposite points of the CMB are seperated by $\sim 1.96\, d_h$ ∴ causally disconnected. They haven't had time to be in thermal contact with each other yet they have the same T to within 10^{-5}.

Another way: the horizon distance @ t_{ls} is

$$d_{hor}(ls) = \frac{2c}{H(ls)} \simeq 0.4 \text{ Mpc}.$$

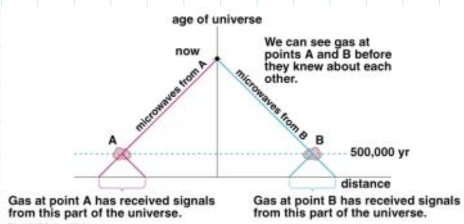
The ang. diameter distance to the last scattering surface is $\simeq 13$ Mpc

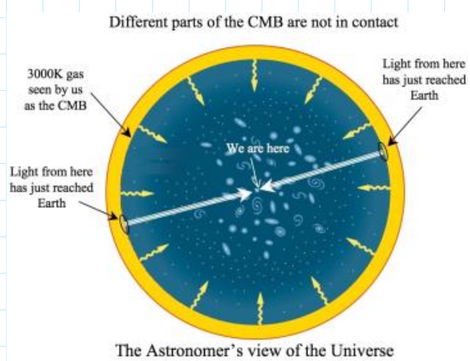
So, seperations on the CMB bigger than

$$\theta_{hor} = \frac{0.4 \text{ Mpc}}{13 \text{ Mpc}} \simeq 2^\circ$$

are not causally connected.

Yet, have the same T to within 10^{-5} !





2) Flatness Problem

Recall: FE written as $1-\Omega(t) = -\frac{Kc^2}{H^2a^2}$

At the present time: $1-\Omega_0 = \frac{-Kc^2}{H_0^2 a_0^2}$

CMB + SN Ia tell us $|1-\Omega_0| \leq 0.2$

If $\Omega_0 \simeq 1$ now, what does it have to be in the past.

Look at $1-\Omega(t) = \frac{H_0^2(1-\Omega_0)}{H^2a^2}$

For a matter dom. Univ.

$a \propto t^{2/3}$

$\rightarrow \dot{a} \propto t^{-1/3}$

$\rightarrow \frac{\dot{a}}{a} \propto t^{-1}$ $\rightarrow H^2 = \left(\frac{\dot{a}}{a}\right)^2 \propto t^{-2}$

$\therefore H^2a^2 \propto t^{-2}t^{4/3} \propto t^{-2/3}$

For a rad'n dom. Univ. $H^2a^2 \propto t^{-1}$

When matter dominated $|1-\Omega|_m \propto t^{2/3}$

" rad'n dominated $|1-\Omega_r| \propto t$

So, any deviation from 1 grows with time!

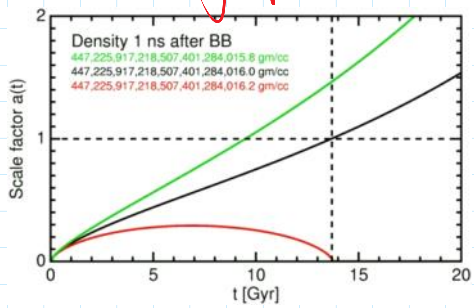
if $|1-\Omega_0| \leq 0.2$, then at the time of radiation-matter equality

$$|1-\Omega|_{rm} \leq 2\times10^{-4}$$

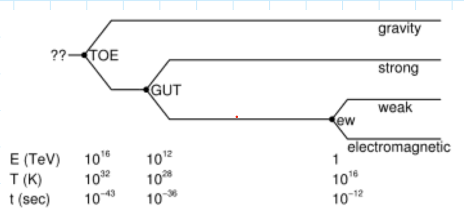
@ BBN $|1-\Omega|$ $\leq 3\times10^{-14}$

@ BBN $|1-\Omega|_{bbn} \leq 3 \times 10^{-14}$

The Univ. is very close to flat now so must have been extremely, preciesly flat @ early times. 'Fine-tuning problem'.



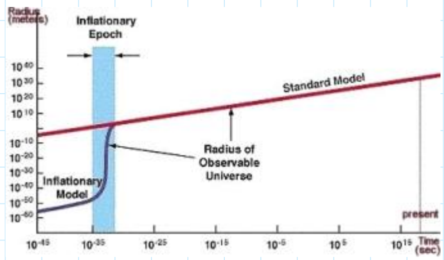
3) Magnetic Monopole (or relic particle) problem



GUT theories predice extremely massive particles ($\sim 10^{16}$ GeV) such as magnetic monopoles. Densities @ time of creation would be $\sim 10^{82}\,m^{-3}$ but non-relativistic, so Univ. would be matter dominated very rapidly. But we don't see any mag. monopoles at all (one of the original motivations of inflation)

Inflation: Guth (1981)

Inflation $\Leftrightarrow \ddot{a}(t) > 0$



So, acc'n eq'n: $\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}\left(\rho + \frac{3P}{c^2}\right)$

$\rightarrow \rho c^2 + 3P < 0$

or $P < -\frac{\rho c^2}{3}$; ie, negative pressure

Classic example of inflationary expansion is a Univ. w/ a cosmo. constant. This is equiv. to having a fluid w/ $P = -\rho c^2$

Alread saw that a Λ-dominated expansion gives $a(t) = e^{\sqrt{\frac{\Lambda}{3}}\,t}$ $\therefore$ $H = \frac{\dot{a}}{a} = \left(\frac{\Lambda}{3}\right)^{1/2}$ and $a \propto e^{Ht}$

After some time, inflation must come to an end, w/ the energy in Λ being converted into conventional matter. Should think of this as a decay of the particles acting as Λ into normal particles. If all of this happens at $t \sim 10^{-35}$ s ($\sim$GUT scale) then none of the successes of the BB theory are lost.

How Inflation Solves the Flatness Problem

Recall $|\Omega_0(t) - 1| = \frac{|K|c^2}{a^2 H^2}$

$$\frac{\quad}{a^2 H^2}$$

If $\ddot{a} > 0 \Rightarrow \frac{d(\dot{a})}{dt} > 0 \Rightarrow \frac{d}{dt}(aH) > 0$

So, $\Omega_0(t)$ is being driven towards 1. In the case of perfect exponential expansion

$$|\Omega(t) - 1| \propto e^{-\sqrt{\frac{4\Lambda}{3}}\,t}$$

Size of the Univ that can be observed is $\sim cH^{-1}$. If this expansion happens so fast that cH^{-1} is unchanged (or changes very little) than the observed curvature vanishes.