How Inflation Solves the Horizon Problem

At any time t, the horizon distance is given by $d_h(t) = a(t)\, c \int_0^t \frac{dt}{a(t)}$

Prior to inflation, the Univ. was rad'n dom. Thus, at the beginning of inflation, the horizon dist. was $d_h(t_i) = a_i c \int_0^{t_i} \frac{dt}{a_i (t/t_i)^{1/2}} = 2ct_i$

At the end of inflation:

$$d_h(t_f) = (a_i e^N)\, c \left(\int_0^{t_i} \frac{dt}{a_i (t/t_i)^{1/2}} + \int_{t_i}^{t_f} \frac{dt}{a_i e^{(H_i(t-t_i))}} \right)$$

Look @ 2nd integral:

$$\int_{a_i}^{a_f} \frac{da}{\dot{a}a} = \int_{a}^{a_f} \frac{da}{H_i a^2} \simeq \frac{1}{H_i a_i} \quad \text{if } N \text{ is large}$$

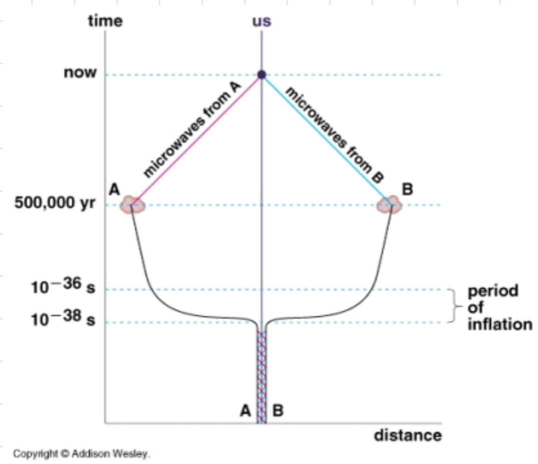
$$\therefore\ d_h(t_f) = e^N c\,(2t_i + H_i^{-1})$$

If inflation started at $t_i \sim 10^{-36}$ s, $H_i^{-1} \sim 10^{-36}$ s & $N \sim 100$, then

$$d_h(t_i) = 2ct_i = 6 \times 10^{-28}\ \text{m}$$

$$d_h(t_f) \simeq e^N 3ct_i \simeq 2 \times 10^{16}\ \text{m} \simeq 0.8\ \text{pc}$$

$\therefore$ Each small patch of the Univ. which was in thermal equil. before can expand to be enormous. ∴ Each patch of the CMB was in thermal contact.



Solving the Monopole Problem

If # is conserved, then during inflation

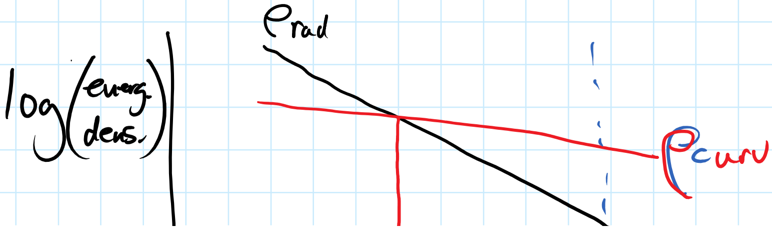
$n \propto e^{-3H_i t}$, so after 100 e-foldings

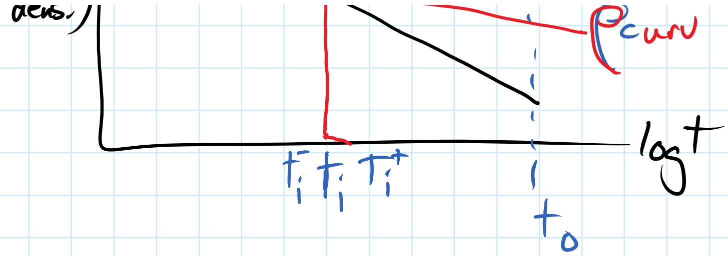
$n \propto e^{-300}$, v.v. small

How Much Inflation?

$$\frac{\dot{a}^2}{a^2} = \frac{8\pi G \rho}{3} - \frac{Kc^2}{a^2} = \frac{8\pi G}{3}\Big(\rho - \overbrace{\frac{3Kc^2}{8\pi G a^2}}^{\rho_{curv}}\Big)$$

so $\rho_{curv} \propto \frac{1}{a^2}$ and know $\rho_{rad} \propto \frac{1}{a^4}$





Suppose at time t_i, radiation energy dens. and curvature energy density are the same.

$$\rho_{curv} = \frac{3Kc^2}{8\pi G a_i^2} = \rho_\gamma = \frac{\rho_0 a_0^4}{a_i^4}$$

$$\text{ie. } \rho_0 = \frac{3Kc^2}{8\pi G} \frac{a_i^2}{a_0^4}$$

in absence of inflation

$$\left.\frac{\rho_{curv}}{\rho_\gamma}\right|_{now} = \frac{\frac{3Kc^2}{8\pi G a_0^2}}{\rho_0} = \left(\frac{a_0}{a_i}\right)^2 \simeq \left(\frac{t_0}{t_i}\right)$$

assuming $t^{1/2}$ growth of a

$$\text{so } \left.\frac{\rho_{curv}}{\rho_\gamma}\right|_{now} \sim \left(\frac{10^{17}\,s}{10^{-35}\,s}\right) \sim 10^{52}$$

If inflation occured at $t = t_i \sim 10^{-35}$ s

$$@\ t_i^- \quad \rho_{curv} = \frac{3Kc^2}{8\pi G a_{i^-}^2}, \quad \rho_\gamma = \frac{\rho_0 a_0^4}{a_{i^-}^4}$$

$$@\ t_i^+ \quad \rho_{curv} = \frac{3Kc^2}{8\pi G a_{i+}^2}, \quad \rho_\gamma = \frac{\rho_0' a_0^4}{a_{i+}^4}$$

$\quad a_i^+ \quad a_i^+$

where $\rho_0' = \left(\frac{a_i^+}{a_i^-}\right)^2 \rho_0$

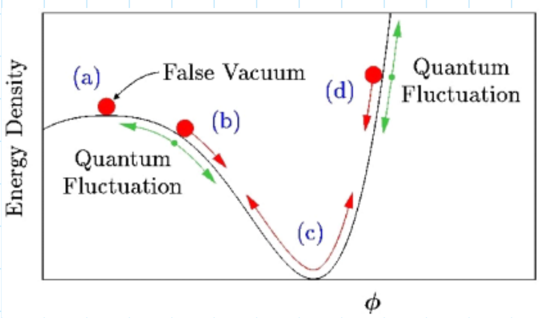
so that $\rho_r(t_i^-) = \rho_r(t_i^+)$ due to reheating

then as long as $\frac{\rho_{curv}(t_i^+)}{\rho_{curv}(t_i^-)} = \left(\frac{a_i^-}{a_i^+}\right)^2 < 10^{-52}$

then $\frac{\rho_{curv}}{\rho_r}$ will be ≤ 1 now, ie. $\frac{a_i^+}{a_i^-} \geq 10^{26}$

This can all happen very fast, say, b/w 10^{-36} & 10^{-34} s : $\frac{a_{final}}{a_{initial}} \simeq e^{[H(t_f - t_i)]} = e^{99}$

$\simeq 10^{43}$



Scaler field dominates energy but decays quickly.

Evidence for Inflation in the CMB

Links to Sean Carroll's blog:

http://www.preposterousuniverse.com/blog/2014/03/16/gravitational-waves-in-the-cosmic-microwave-background/

http://www.preposterousuniverse.com/blog/2014/03/16/bicep2-updates/

http://www.preposterousuniverse.com/blog/2014/09/21/planck-speaks-bad-news-for-primordial-gravitational-waves/

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