An Expanding Universe

In 1929, Hubble measured redshifts z from ~20 galaxies. And then estimated distances to them.

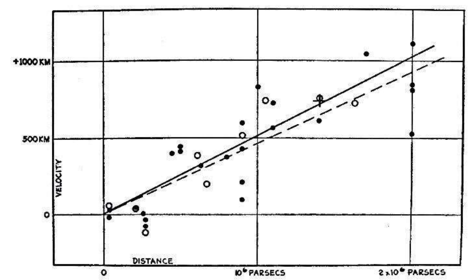


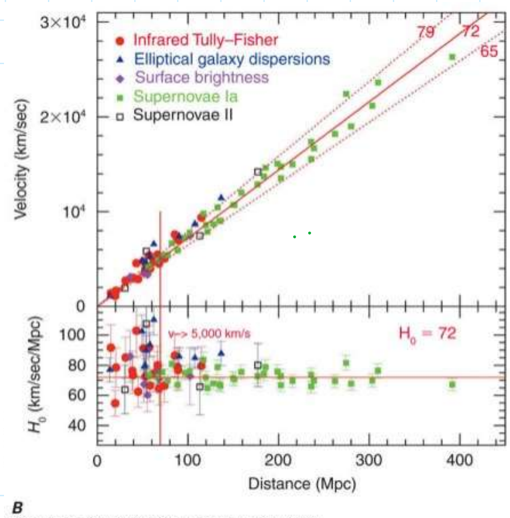
Figure 2.4: Edwin Hubble's original plot of the relation between redshift (vertical axis) and distance (horizontal axis). Note that the vertical axis actually plots cz rather than z – and that the units are accidentally written as km rather than km/s. (from Hubble 1929, Proc. Nat. Acad. Sci., 15, 168)

He found $cz = H_0 r$

where H_0 is a constant (called the Hubble constant)

$$[H_0] = \left[\frac{1}{s}\right]$$

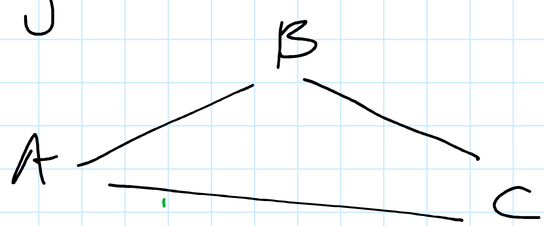
(km/s/Mpc)



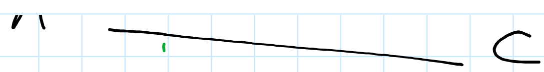
(Wendy L. Freedman, Observatories of the Carnegie Institution of Washington, and NASA)

This is exactly what you would expect to see in a Universe undergoing homogeneous and isotropic expansion.

Consider 3 galaxies



After homo. and iso. expansion



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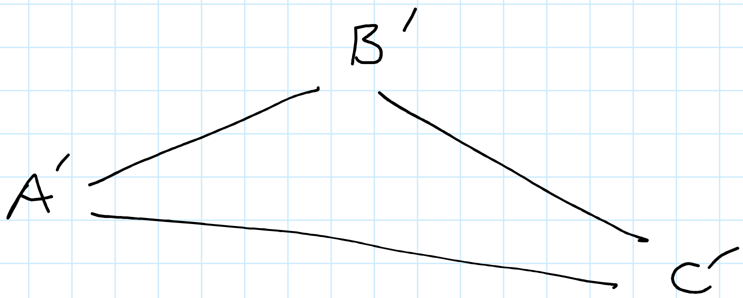


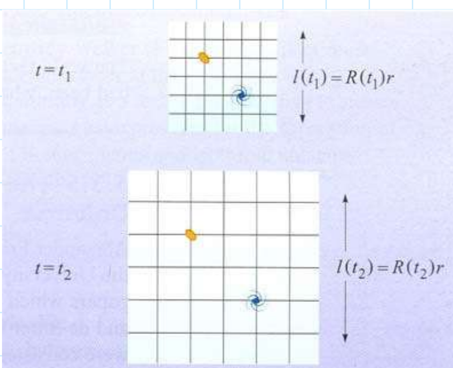
Each side expanded by a constant factor:

$a(t) =$ ale factor

Distance from $A' \to B'$

$$d_{A'B'} = a(t)\, l_{AB}$$

l_{AB} = co-moving distance (fixed)



$$\frac{d(d_{A'B'})}{dt} = \frac{da}{dt}\, l_{AB}$$

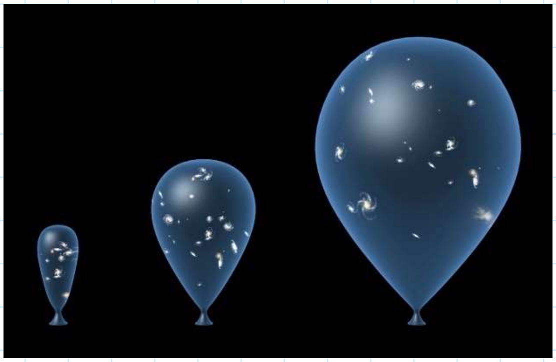
or $V = \dot{a}\, l_{AB}$

but $l_{AB} = \dfrac{d_{A'B'}}{a(t)} \to$ $V = \left(\dfrac{\dot{a}}{a}\right) d$

$\boxed{V = Hd}$ Hubble parameter

$H = \dfrac{\dot{a}}{}$

$$H = \frac{\dot{a}}{a}$$



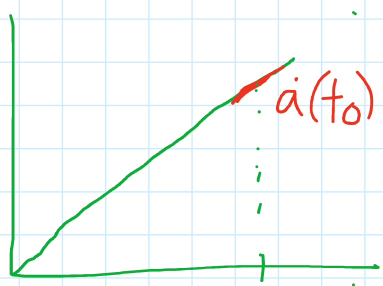
Expanding Univ. means Big Bang!

For small v ($v \ll c$), $\frac{\Delta\lambda}{\lambda} = \frac{v}{c}$

or $v = c\left(\frac{\Delta\lambda}{\lambda}\right) = cz$ where

$z = \text{redshift} = \frac{\Delta\lambda}{\lambda}$

If $\dot{a}(t) = \text{const.}$, then Hubble parameter now (H_0) gives age of Univ.



$\dot{a}(t_0) = \frac{a_0}{t_0}$; $\frac{\dot{a}_0}{a_0} = H_0 = \frac{1}{t_0}$

$H_0 = 70\ \text{km s}^{-1}\ \text{Mpc}^{-1}$

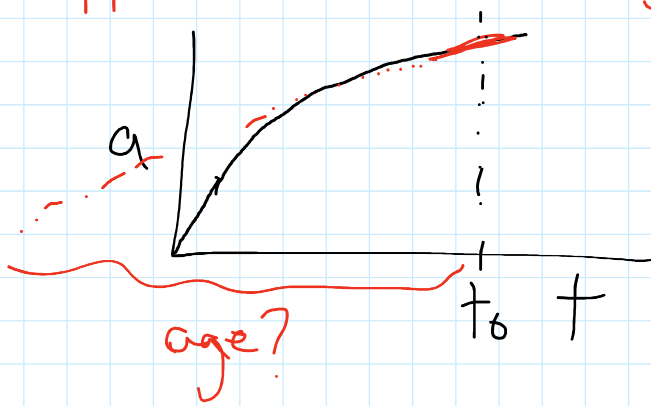
so $\frac{1}{H_0} \simeq t_0 = 14\ \text{Gyr}$

'Hubble time'

Similarly, Hubble distance

$$\frac{c}{H_0} \simeq 4300 \text{ Mpc}$$

If the Univ. accelerates in any way (e.g. due to matter) then $1/H_0$ only gives an upper-limit to the age:



$$\frac{\dot{a}_0}{a_0} = H_0 < \frac{1}{t_0}$$

$$\hookrightarrow t_0 < \frac{1}{H_0}$$

An Expanding Universe

In 1929, Hubble measured redshifts, z, from ~20 galaxies and estimated distances to them. He found

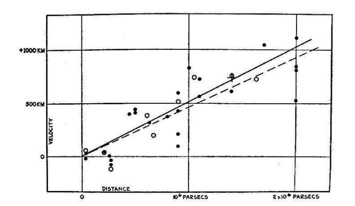


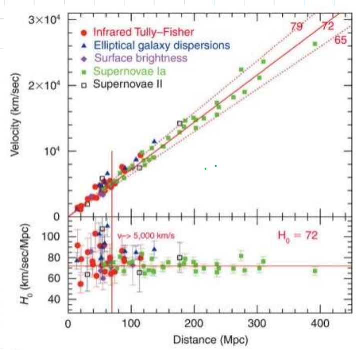
Figure 2.4: Edwin Hubble's original plot of the relation between redshift (vertical axis) and distance (horizontal axis). Note that the vertical axis actually plots cz rather than z – and that the units are accidentally written as km rather than km/s. (from Hubble 1929, Proc. Nat. Acad. Sci., 15, 168)

$$cz = H_0 d$$

where H_0 is a constant (called the Hubble constant)

$$[H_0] = \left[\frac{1}{s}\right] \quad (\mathrm{Km/s/Mpc})$$

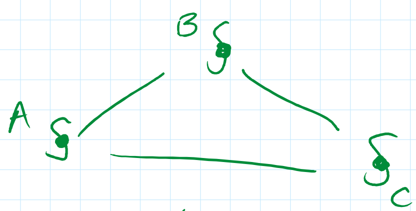
This is exactly what you would expect to see in a Universe undergoing homo. and isotropic expansion.



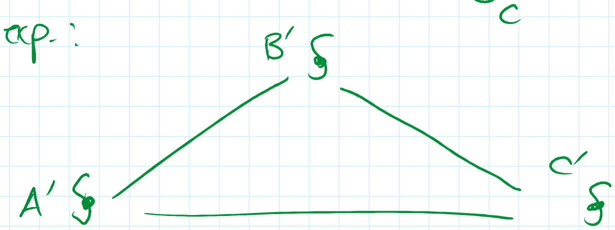
B

(Wendy L. Freedman, Observatories of the Carnegie Institution of Washington, and NASA)

Consider 3 galaxies:



After iso. & homo. exp.:



Each side has expanded by a const. factor:

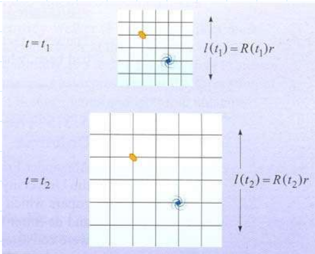
$a(t)$ = scale factor

Distance from $A' \to B'$: $d_{A'B'} = a(t)\, l_{AB}$

l_{AB} = co-moving distance (fixed)

$d(d_{A'B'}) = da\, l_{AB}$ or $v = \dot{a}\, l_{AB}$

$\frac{}{dt}$ $\frac{}{dt}$ l_{AB} or $v = \dot{a}\, l_{AB}$



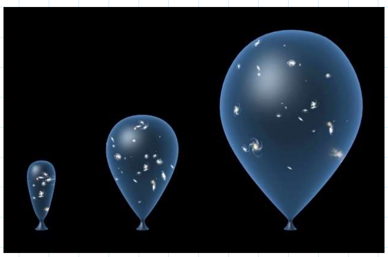
but $l_{AB} = \frac{d_{A'B'}}{a}$

so $\boxed{V = \boxed{\left(\frac{\dot{a}}{a}\right)} d}$ → Hubble parameter

$$H(t) = \frac{\dot{a}(t)}{a(t)}$$

$$\therefore \; V = Hd$$

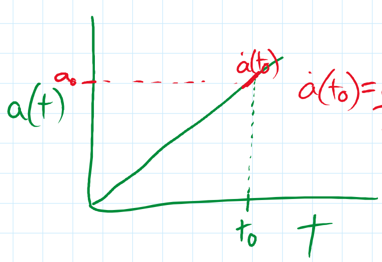
$H_0 = \frac{\dot{a}_0}{a_0}$ ← now or evaluated at current time



For small V, $\frac{\Delta\lambda}{\lambda} = \frac{V}{c}$ or $V = cz$

where z = redshift = $\frac{\Delta\lambda}{\lambda}$

If $\dot{a}(t)$ = constant, then Hubble parameter now (H_0) gives age of Univ.

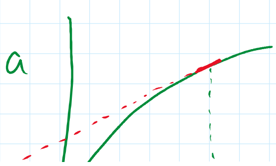


$\dot{a}(t_0) = \frac{a_0}{t_0}$, $\frac{\dot{a}_0}{a_0} = \frac{1}{t_0} = H_0$

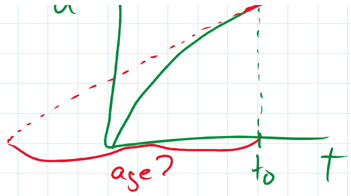
$H_0 = 70$ Km/s/Mpc

$\frac{1}{H_0} \simeq 14$ Gyr 'Hubble time'

If the Univ. accelerates in any way (due to matter) then H_0 only gives a limit to the age



$\frac{\dot{a}_0}{a_0} = H_0 < \frac{1}{t_0}$ get an upperlimit

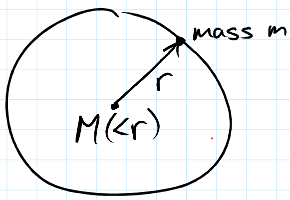


$$\frac{\dot a_0}{a_0} = H_0 < \frac{1}{t_0}$$ get an upperlimit

['Hubble distance': $\frac{c}{H_0} \simeq 4300$ Mpc]
est. of horizon dist.

Newtonian Cosmology

Consider an observer in an uniform expanding medium w/ mass density ρ. Since the Universe looks the same from anywhere we can consider any point to be its center.



Consider a particle a distance r away w/ mass m

$$F = m\dot v = m\ddot r$$

in exp. Univ. (homo & iso)

$$r = a x \qquad x\text{: comoving dist}$$

$$\ddot r = \ddot a x$$

$$\therefore \; m\ddot a x = -\frac{G m M(<r)}{a^2 x^2}$$ (no grav. force from outside shell)

$$M(<r) = \frac{4}{3}\pi r^3 \rho(t) = \frac{4}{3}\pi a^3 x^3 \rho(t)$$

$$\Rightarrow \ddot a(t) = -\frac{4}{3}\pi G \rho(t) a(t)$$ [Exp. depends only on density!]

Cons. of mass : $\rho(t)\left(a(t)x\right)^3 = \rho(t_0)\left(a(t_0)x\right)^3$

or $\rho(t) = \rho(t_0)\left(\frac{a(t_0)}{a(t)}\right)^3$

Follow convention $a(t_0) = 1$

$$\therefore \; \ddot a(t) = -\frac{4}{3}\pi G \frac{\rho(t_0)}{a^2(t)}$$

$\times$ both sides by $2\dot a(t)$

$$2\dot a \ddot a = -\frac{4}{3}\pi G \rho(t_0) \frac{2\dot a}{a^2}$$

$$\frac{d}{dt}(\dot{a}^2) = +\frac{4}{3}\pi G\rho(t_0)\frac{d}{dt}\left(\frac{1}{a}\right)\cdot 2$$

$$\rightarrow \quad \dot{a}^2 = \frac{8\pi G\rho(t_0)}{3}\frac{1}{a} + \text{const.}$$

$$\therefore \boxed{\dot{a}^2 = \frac{8\pi G\rho a^2}{3} + \text{const.} = \frac{8\pi G\rho a^2}{3} - Kc^2}$$

Friedmann Eqn.

Solving this for $a(t)$ tells us about the growth, history & future of the Univ.

Not entirely correct (no GR)

Newtonian interp. (mult. both sides by $\frac{x^2}{2}$)

$$\frac{(\dot{a}x)^2}{2} - \frac{4}{3}\pi(ax)^2\rho G = -Kc^2\frac{x^2}{2}$$

$$\frac{v^2}{2} - \frac{4}{3}\pi r^3\frac{\rho G}{r} = -Kc^2\frac{x^2}{2}$$

$$\frac{v^2}{2} - \frac{GM}{r} = -Kc^2\frac{x^2}{2}$$

$$\left(\frac{KE}{\text{mass}}\right) + \left(\frac{PE}{\text{mass}}\right) = -\frac{Kc^2x^2}{2} = \text{const.}$$

Cons. of energy

$K \propto$ energy of a comoving particle and history of exp. depends on K