Statistical Properties of Density Fluctuations

Define the correlation function. Since galaxies are not randomly distributed, the probability of finding a galaxy at $\vec{x}$ is not independent of whether there is a galaxy in the vicinity of $\vec{x}$. It is more probable to find a galaxy in the vicinity of another one than at an arbitrary location.

Let $\bar{n}$ = avg. number density of galaxies

Then the probability of finding a galaxy in a volume element dV is $P_1 = \bar{n}\, dV$

(independent of position if Univ. is homogeneous ; dV is chosen such that $P_1 \ll 1$)

Now, the probability of finding a galaxy in dV at location $\vec{x}$ and, at the same time, finding a galaxy in dV at $\vec{y}$ is

$$P_2 = (\bar{n}\, dV)^2 \left[1 + \xi_g(\vec{x}, \vec{y}) \right]$$

where $\xi_g(\vec{x}, \vec{y})$ is the 2-pt. correlation function.

By analogy, can define ξ for the total matter density $\langle \rho(\vec{x})\rho(\vec{y}) \rangle = \bar{\rho}^2 \langle [1+\delta(\vec{x})][1+\delta(\vec{y})] \rangle$

$$= \bar{\rho}^2 \left(1 + \langle \delta(\vec{x})\delta(\vec{y}) \rangle\right)$$

b/c $\langle \delta(\vec{x}) \rangle = 0$ for all $\vec{x}$ $\longrightarrow$

$$\equiv \bar{\rho}^2 \left(1 + \xi(\vec{x}, \vec{y})\right)$$

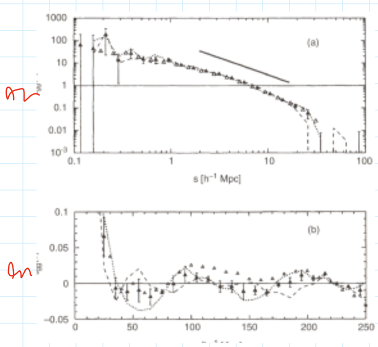
$\xi(\vec{x}, \vec{y})$ can be simpilfied using the cosmo. principle:

homogeneous means $\xi(\vec{x}, \vec{y}) \to \xi(\vec{x} - \vec{y})$

isotropy means $\xi(\vec{x} - \vec{y}) \to \xi(|\vec{x} - \vec{y}|) = \xi(r)$

For a homogeneous random field,

ξ can be determined by averaging over the density products for a large # of pairs with given seperation r.



For galaxies, find the approximate relation $\xi_g(r) = \left(\frac{r}{r_0}\right)^{-\gamma}$ where

$r_0 \simeq 5\ h^{-1}$ Mpc is the correlation length and $\gamma \simeq 1.8$ (valid for $2h^{-1}\text{Mpc} < r < 30\ h^{-1}\text{Mpc}$)

ξ and $P(K)$ are related by

$$P(K) = 2\pi \int_0^\infty dr\, r^2 \frac{\sin Kr}{Kr} \xi(r)$$

Can be inverted

Evolution of Density Fluctuations

$P(K)$ & $\xi(r)$ both depend on time as structure evolves. If we interpret $\vec{x}$ as a comoving seperation vector, then since $\delta(\vec{x},t) = D_+(t)\,\delta_0(\vec{x})$

$$\xi(x,t) = D_+^2(t)\,\xi(x,t_0)$$

and $P(K,t) = D_+^2(t) P(K,t_0) \equiv D_+^2(t) P_0(K)$

In the <u>linear regime</u>.

This isn't right in the rad'n dominated regime, so another function, called the <u>transfer function</u> is needed: $P(K,t) = D_+^2(t)\,T^2(K)\,P_0(K)$

$T(K)$ depends on the cosmo model & type of DM.

The Shape & Norm. of $P(K)$

The changing shape of $P(K)$ is encoded by $T(K)$. In linear perturbation theory, fluctuation on all K grow independently of each other and can be considered individually.

Consider CDM perturbations w/ a comoving scale L longer than the co-moving horizon $r_{h,com}$

Only for $z < z_{enter}(L)$ does the horizon become larger than L. $z_{enter}(L)$ is the redshift at which the comoving horizon = comoving L

ie. $r_{h,com}(z_{enter}(L)) = L$.

Say that at $z_{enter}(L)$, the perturbation under consideration 'enters the horizon' although really its the other way around.

How a perturbation grows at $z > z_{enter}$ depends on z_{enter}. If $z_{enter}(L) \gtrsim z_{rm}$ (ie, rad'n dom.), then δ can not grow. If $z_{enter}(L) \lesssim z_{rm}$, $\delta \propto D_+(t)$

Thus, the length scale

$$L_0 = r_{h,com}(z_{rm}) = \frac{c}{\sqrt{2}\, H_0} \frac{1}{\sqrt{(1+z_{rm}\, \Omega_{m,0})}}$$

$$\simeq 12 \left(\Omega_{m,0} h^2\right)^{-1} \mathrm{Mpc}$$

is critical to how fluctuations grow

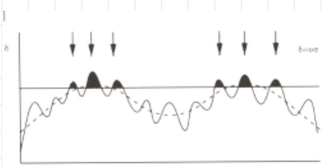
Fluctuations w/ $L > L_0$ enter the horizon after matter domination and can grow. Fluctuations w/ $L < L_0$ enter at rad'n dom. & cannot grow until z_{rm}. Thus, their relative growth up to the present time is smaller than those w/ $L > L_0$

Treating the growth of all of these scales gives

$T(K)$.

$T(K)$ depends on KL_0 ; thus $K(\Omega_{m,0}h^2)^{-1}$.
Since distance determined from z are measured in h^{-1} Mpc, the shape of $T(K)$ and hence $P(K)$ depends on $\Gamma = \Omega_{m,0}h$ = shape parameter

The normalization of $P(K)$ is typically done using the galaxy distribution, but galaxies are thought to be a biased tracer of the DM dist'n: $\delta_g \equiv \frac{\Delta n}{\bar{n}} = b\frac{\Delta \rho}{\bar{\rho}} = b\delta$



where $\bar{n}$ is avg. density of the galaxy pop. ; $\Delta n = n - \bar{n}$ is the deviation of the local n from the avg. density.