Light beams are subject to differential deflection
∴ The solid angle which the image of the source subtends on the sky changes. Since surface brightness is preserved, the flux = SB × SA will change by

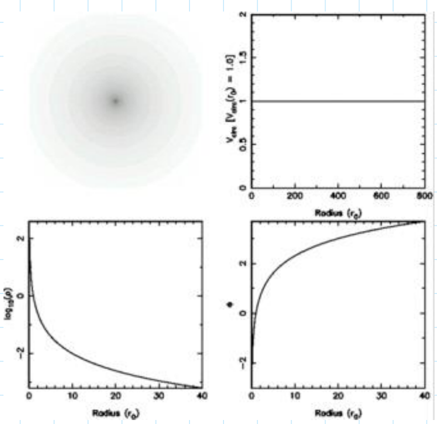
$$\mu = \frac{S}{S_0} = \frac{\omega}{\omega_s}$$; if $\mu > 1$, get magnification.

For sources that are small compared to the characteristic

$$\mu = \left| \det\left(\frac{d\vec{\beta}}{d\vec{\theta}}\right) \right|^{-1}$$

Example: Isothermal Sphere

A mass model for a galaxy that gives a flat rotation curve.



$$\rho(r) = \frac{\sigma_v^2}{2\pi G r^2}$$ where σ_v is the 1D vel disp. of stars if the dist'n of stellar orbits is isotropic.

Projecting this onto a flat sheet yields

$$\Sigma(\xi) = \frac{\sigma_v^2}{2G\xi}$$

So that the projected mass $M(\xi)$ within ξ

$$M(\xi) = 2\pi \int_0^{\xi} \xi' \Sigma(\xi')\, d\xi' = \frac{\pi \sigma_v^2 \xi}{G}$$

Since this is axially symmetric and

$$\vec{\hat{\alpha}} = \frac{4GM(\xi)}{c^2 \xi} \quad \text{get} \quad \hat{\alpha}(\xi) = 4\pi \left(\frac{\sigma_v}{c}\right)^2$$

$$\Rightarrow \alpha(\theta) = 4\pi \left(\frac{\sigma_v}{c}\right)^2 \left(\frac{D_{ds}}{D_s}\right) \equiv \theta_E$$

$\therefore$ the deflection angle is constant & equals θ_E and depends quadratically on σ_v.

The charactistic scale of $\theta_E = 1''15 \left(\frac{\sigma_v}{200 \frac{km}{s}}\right)^2 \left(\frac{D_{ds}}{D_s}\right)$

Lens eq'n: $\beta = \theta - \theta_E \frac{\theta}{|\theta|}$

If $|\beta| < \theta_E$, 2 soln's of the lens eq'n:

$$\theta_1 = \beta + \theta_E \quad ; \quad \theta_2 = \beta - \theta_E$$

If we assume $\beta \geq 0$ then $\theta_1 > \theta_E > 0$ & $0 > \theta_2 > -\theta_E$ $\therefore$ one image is located on either side of the lens w/ $\Delta\theta = \theta_1 - \theta_2 = 2\theta_E$

$$= 2''3 \left(\frac{\sigma_v}{200 \frac{km}{s}}\right)^2 \left(\frac{D_{ds}}{D_s}\right);$$

does not depend on the pos'n of the source (β)

For $\beta > \theta_E$, only one image exists at θ_1

$$\mu(\theta) = |\theta / \theta_E| \qquad \text{if } \theta \simeq \theta_E, \mu \text{ is}$$

$$\mu(\theta) = \frac{|\theta/\theta_E|}{\left| |\theta/\theta_E| - 1 \right|}$$ if $\theta \simeq \theta_E$, μ is very very large

Luminosity Function of Galaxies

The LF is the # density of agalaxies (of a certain type or all types) at a specific lumin.

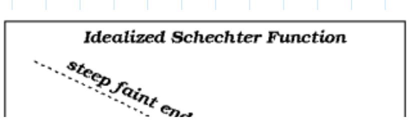
$\Phi(L)\,dL$ is defined as the # density of galaxies b/w L & $L+dL$.

The global galaxy LF is well approximated by the Schechter LF: $\Phi(L) = \left(\frac{\Phi^*}{L^*}\right)\left(\frac{L}{L^*}\right)^{\alpha} e^{-L/L^*}$

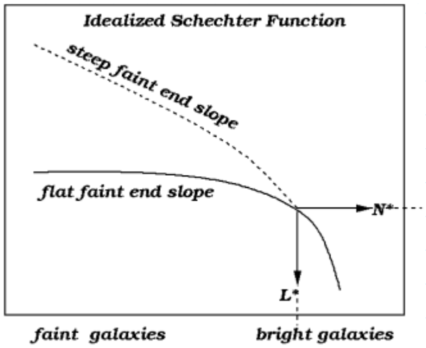
where L^* is a characteristic luminosity above which the distribution decreases exponentially. α is the small L slope & Φ^* sets the normalization

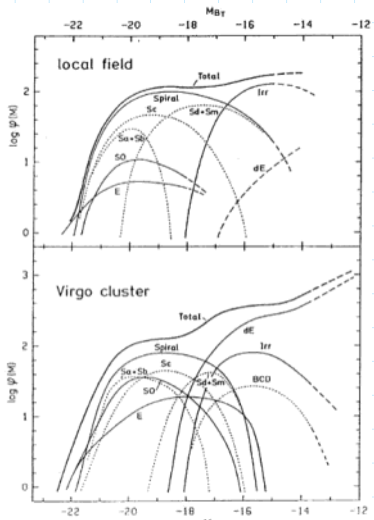
In magnitudes, use $\frac{dL}{L} = -0.4 \ln 10\, dM$ and $\Phi(L)\,dL = \Phi(M)\,dM$ one obtains

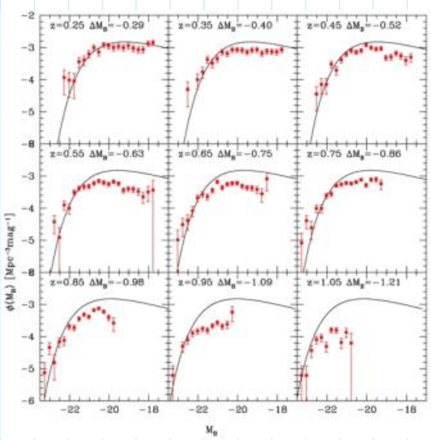
$$\Phi(M) = (0.4 \ln 10)\, \Phi^* \, 10^{0.4(\alpha+1)(M^*-M)} e^{\left(-10^{0.4(M^*-M)}\right)}$$



Typically, $\alpha \simeq -1$ & $\Phi^* \simeq 1.6 \times 10^{-2}\, h^3\, Mpc^{-3}$







Typically, $\alpha \simeq -1$; $\Phi^* \simeq 1.6\times10^{-2}\, h^{3}\,\mathrm{Mpc}^{-3}$
and $L^* \sim 1\times10^{10}\, h^{-2} L_\odot$

The total # density of galaxies is formally infinite if $\alpha \leq -1$, but galaxies are not found at arbitrarily small L.

The luminosity density $l_{tot} = \int_0^\infty dL\, L\, \Phi(L)$

$$= \Phi^* L^* \Gamma(2+\alpha)$$

is finite if $\alpha \geq -2$ ($\Gamma(x)$ is the Gamma function which, for positive integers, $\Gamma(n+1) = n!$)

∴ $l_{tot} \sim \Phi^* L^*$ is a good approx to the lumin density.

The # density $n(>L) = \Phi^* \Gamma(\alpha+1, L/L^*)$
; for $\alpha = -1.5$, about ½ of the galaxies are brighter than $0.7L^*$. ∴ L^* is usually used as a typical galaxy luminosity.

- LFs are different for diff. types of galaxies, environments ; different wavelengths

- LFs evolve w/ z.