Review

Cosmological Principle: On large scales Universe is homogeneous & isotropic. (e.g. CMB)

Combining this w/ observations of an expanding Universe - Yields the Roberston-Walker metric:

$$ds^2 = -c^2 dt^2 + a^2(t)\left[\frac{dr^2}{1-Kr^2} + r^2\left(d\theta^2 + \sin^2\theta\, d\phi^2\right)\right]$$

where $a(t)$ = scale factor

K = const. which meas. curv. of space

r, θ, ϕ = comoving coordinates

Solving the field equations w/ this metric gives the Friedmann eqn

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G\rho}{3} - \frac{Kc^2}{a^2} + \frac{\Lambda}{3}$$

Λ = cosmo. constant

acc'n eq'n: $$\left(\frac{\ddot{a}}{a}\right) + \frac{4\pi G}{3}\left[\rho + \frac{3p}{c^2}\right] - \frac{\Lambda}{3} = 0$$

To determine the expansion history of the Univ. need the fluid eqn $\dot{\rho} + \frac{3\dot{a}}{a}\left(\rho + \frac{P}{c^2}\right) = 0$

and an eqn of state.

e.g. matter $P = 0 \to \rho \propto a^{-3}$

radiation $P = \frac{\rho c^2}{3} \to \rho \propto a^{-4}$

Cosmological parameters:

$H_0 = \frac{\dot{a}_0}{a_0}$ (more generally, $H = \frac{\dot{a}}{a}$) Hubble const.

$\rho_{c,0} = \frac{3H_0^2}{8\pi G}$ critical density at current time

$\Omega_{m,0} = \frac{\rho_{m,0}}{\rho_{c,0}}$, $\Omega_m = \frac{\rho_m}{\rho_c}$, $\Omega_{rad} = \frac{\rho_{rad}}{\rho_c}$, etc.

$q_0 = -\frac{\ddot{a}_0 a_0}{\dot{a}_0^2}$ acc'n parameter, $\Omega_0 = \sum_i \Omega_{i,0}$

$\therefore \; Kc^2 = \frac{8\pi G}{3\Omega_0}\left[\rho_0(\Omega_0 - 1)\right]$ $(\Lambda = 0)$

K is determined by energy content of the Univ. (including Λ)

$K < 0$, Univ. is -vely curved

$K = 0$, Univ. is flat

$K > 0$, Univ. is +vely curved

- redshift: $1+z = \dfrac{a_0}{a}$ where $z = \dfrac{\Delta\lambda}{\lambda}$

- Proper distance $\ell = a_0 \displaystyle\int_0^{r_0} \frac{dr}{\sqrt{1-Kr^2}} = a_0 f(r_0)$

- For light distance set $ds = 0$ obtain $r_0 = r_0(z)$ from $\displaystyle\int_0^{r_0} \frac{dr}{\sqrt{1-Kr^2}} = \int_{t_1}^{t_0} \frac{c\,dt}{a(t)}$

- define different distances

$$f = \frac{L}{4\pi d_{lum}^2} \qquad d_{lum} = r_0 a_0 (1+z)$$

$$\theta = \frac{D}{d_{ang}} \; ; \; d_{ang} = \frac{r_0 a_0}{(1+z)} \rightarrow d_{ang} = \frac{d_{lum}}{(1+z)^2}$$

- all distances about the same when $z \ll 1$

- define horizon distance: d_H = that part of the Univ. in casaul contact = distance light has travelled during the current age of the Univ.

Early Univ.

- persuasive arguments for an inflationary ($\ddot{a} > 0$) epoch; solves the horizon, flatness & monopole problems

- rad. dom. epoch until $a_{rm} = \frac{1}{24{,}000\,\Omega_{m,0} h^2}$ $\simeq 50{,}000$ yrs

- but $T \propto \frac{1}{a}$ so T is high enough to keep baryons ionized until $z \simeq 1370$

- photons decouple at $z = 1100 \rightarrow$ produce CMB

  $T_0 = 2.725 \pm 0.001\,K$

- $\frac{n_{bary}}{n_{phot}} = 6 \times 10^{-10} = \eta$

- Power spect. of CMB gives strong constraints on Univ. geom → flat ∴ $\Omega_0 \simeq 1$, but obs. $\Omega_{m,0} \simeq 0.3$ ∴ Dark energy $\Omega_\Lambda \simeq 0.7$ (also agrees w/ SN Ia measurements)
- BBN at $t \sim 1$ s → a couple of minutes $\Omega_b \simeq 0.04$ so since $\Omega_m \simeq 0.3$, DM is non-baryonic