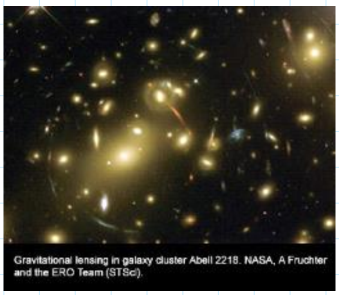
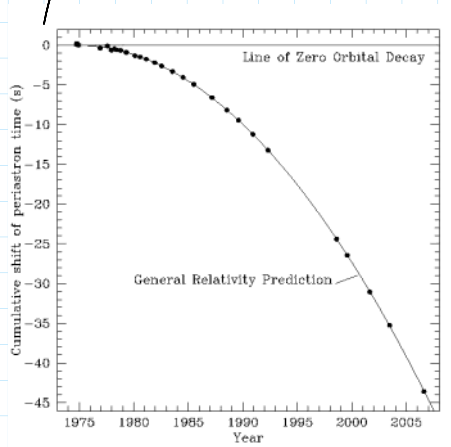
Relativistic Cosmology

G.R.: - predicted the exact perihelion shift of Mercury (43"/century of the 574"/century)

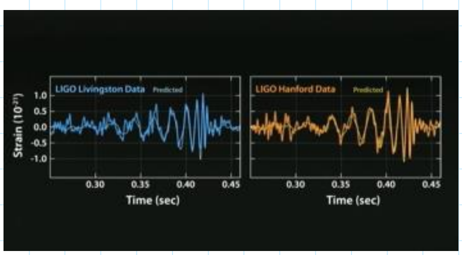
- predicted light will be deflected by gravity



Gravitational lensing in galaxy cluster Abell 2218. NASA, A Fruchter and the ERO Team (STScI).



- gravitional waves



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Differences b/w Newtonian Grav. & GR

1) GR is based on 10 potentials, not 1
2) GR is a non-linear theory, ie. total effect of several bodies is not the simple sum of seperate effects
3) Pressure is 'natively' a source of gravitation as well as density
4) GR usually expressed in terms of geometry;

the 10 potentials give the metric of space-time

The Way of Newton:

Mass tell gravity how to exert a force ($F = -GMm/r^2$)

Force tells mass how to accelerate ($F = ma$)

The Way of Einstein:

Mass-energy tells space-time how to curve

Curved space-time tells mass-energy how to move.

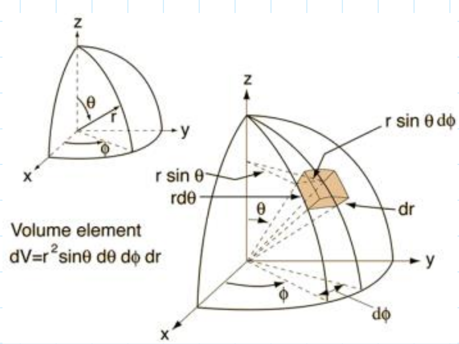
Spacetime characterized by metric

In SR, let x, y, z denote the Cartesian coordinate system and t is the time:

Let 2 neighbouring points in space-time be x, y, z, t and x+dx, y+dy, z+dz, t+dt

$$ds^2 = -c^2dt^2 + dx^2 + dy^2 + dz^2 = -c^2dt^2 + dl^2$$

If $ds = 0$, then $\frac{dl}{dt} = c$; ie, set $ds = 0$ to describe behavior of light



Volume element
$dV = r^2 \sin\theta \, d\theta \, d\phi \, dr$

In spherical polar coordinates,

$x = r\sin\theta\cos\phi$

$y = r\sin\theta\sin\phi$

$z = r\cos\theta$

$$dl^2 = dr^2 + r^2d\theta^2 + r^2\sin^2\theta d\phi^2$$

is SR, $ds^2 = -c^2dt^2 + dr^2 + r^2d\theta^2 + r^2\sin^2\theta d\phi^2$

flat, Euclidean space, Minkowski metric

flat, Eucliden space, MinKowski metric

in GR, $ds^2 = \sum_{\mu,\nu} g_{\mu\nu} dx^\mu dx^\nu$ in general

where $g_{\mu\nu} = \begin{pmatrix} - & - & - & - \\ & - & - & - \\ & & - & - \\ & & & - \end{pmatrix}$ is the metric w/ 10 free parameters/potentials

$\mu, \nu = 0, 1, 2, 3$; x^0 is the time coord. and x^1, x^2, x^3 are the spatial coord's.

We can simpilfy by imposing the cosmo. principle. So, can remove the x-terms and spatial part has a constant curvature

$$dl^2 = \frac{dr^2}{1-Kr^2} + r^2(d\theta^2 + \sin^2\theta\, d\phi^2)$$

Now, add time-dependence to dl to allow space to grow or shrink:

Robertson-Walker Metric

$$ds^2 = -c^2 dt^2 + a^2(t)\left[\frac{dr^2}{1-Kr^2} + r^2(d\theta^2 + \sin^2\theta\, d\phi^2)\right]$$

	SR	GR/RW
Element of radial distance	dr	$\frac{a\,dr}{\sqrt{1-Kr^2}}$
Element of circumference ($\sin\theta = 1$)	$r\,d\phi$	$a r\, d\phi$
Element of area dA	$r^2 \sin\theta\, d\theta\, d\phi$	$a^2 r^2 \sin\theta\, d\phi\, d\theta$
Element of volume dV	$r^2 \sin\theta\, d\theta\, d\phi\, dr$	$\frac{a^3 r^2 \sin\theta\, d\phi\, d\theta\, dr}{\sqrt{1-Kr^2}}$

here, r, θ ; ϕ are comoving coords.

The Proper Distance

The distance to a galaxy far away at $r = r_1$, $\theta = \theta_1$; $\phi = \phi_1$ at time t_1

$$dl = \frac{a\,dr}{\sqrt{1-kr^2}} \Rightarrow l_1 = a(t_1)\int_0^{r_1} \frac{dr}{\sqrt{1-kr^2}} = a(t_1) f(r_1)$$

Compute the same thing at time t_2

$$l_2 = a(t_2) f(r_1)$$

i.e. $$\frac{l_2}{l_1} = \frac{a(t_2)}{a(t_1)}$$

- meaning of $a(t)$ in GR is as a scale factor (same as Newt. case)

- Since r_1 is a comoving coord:

$$\dot{l}_1 = \dot{a}(t_1) f(r_1) = \dot{a}(t_1)\frac{l_1}{a(t_1)} = H(t_1) l_1$$

or, at the current time $V_p(t_0) = H_0 l(t_0)$