Fluid Description of Λ

It is useful to consider a fluid description of Λ. If we define $\rho_\Lambda = \frac{\Lambda}{8\pi G}$, then F.E. is

$$H^2 = \frac{8\pi G}{3}(\rho + \rho_\Lambda) - \frac{Kc^2}{a^2}$$

and define $\Omega_\Lambda = \frac{\rho_\Lambda}{\rho_c}$

Fluid eqn for ρ_Λ is: $\dot{\rho}_\Lambda + \frac{3\dot{a}}{a}\left(\rho_\Lambda + \frac{P_\Lambda}{c^2}\right) = 0$

But, by def'n $\dot{\rho}_\Lambda = 0$

$$P_\Lambda = -\rho_\Lambda c^2 \quad \text{(a negative pressure!)}$$

Can generalize beyond a cosmo. constant. Consider an eqn. of state: $P = w\rho c^2$ where w is a const(?). The case of $w=-1$ is equiv. to Λ, but accelerated expansion can occur if $w < -\frac{1}{3}$ (check this)

Physical cause of Λ could be vacuum energy, but simple estimate of this energy density, its too large by ~124 orders of magnitude.

Matter + Lambda Univ.

Consider a spatially flat Univ. w/ matter & Λ.

$\therefore \Omega_{\Lambda,0} = 1 - \Omega_{m,0}$ where ,0 means a specific time

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F.E. $H^2 = \frac{8\pi G\rho}{3} - \frac{Kc^2}{3} + \frac{\Lambda}{3}$

for matter $\rho = \frac{\rho_0}{a^3}$, $H^2 = \frac{8\pi G\rho_0}{3a^3} + \frac{\Lambda}{3} = \frac{8\pi G\rho_0 H_0^2}{3a^3 H_0^2} + \frac{\Lambda H_0^2}{3H_0^2}$

$$= \left(\frac{\rho_0}{\rho_{c,0}}\right)\frac{H_0^2}{a^3} + \Omega_{\Lambda,0} H_0^2$$

$$= \Omega_{m,0}\frac{H_0^2}{a^3} + (1-\Omega_{m,0}) H_0^2$$

$$\therefore \frac{H^2}{H_0^2} = \frac{\Omega_{m,0}}{a^3} + (1-\Omega_{m,0})$$

A flat Univ. w/ $\Omega_m < 1 \Rightarrow \Omega_\Lambda > 0$ will expand forever.

$$\frac{1}{H_0^2}\frac{\dot{a}^2}{a^2} = \frac{\Omega_{m,0}}{a^3} + (1-\Omega_{m,0})$$

$$\frac{1}{H_0^2}\left(\frac{da}{dt}\right)^2 = \frac{\Omega_{m,0}}{a} + a^2(1-\Omega_{m,0})$$

$$\frac{1}{H_0}\frac{da}{dt} = \sqrt{\frac{\Omega_{m,0}}{a} + a^2(1-\Omega_{m,0})}$$

$$H_0\int_0^t dt = \int_0^a \frac{da}{\sqrt{\frac{\Omega_{m,0}}{a} + a^2(1-\Omega_{m,0})}}$$

if $\Omega_{m,0} > 1$ (or $\Omega_{\Lambda,0} < 0$)

$$H_0 t = \frac{2}{3\sqrt{\Omega_{m,0}-1}} \sin^{-1}\left[\left(\frac{a}{(\Omega_{m,0}/\Omega_{m,0}-1)^{1/3}}\right)^{3/2}\right]$$

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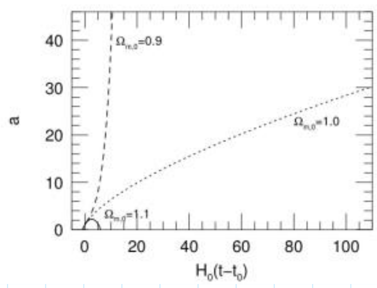


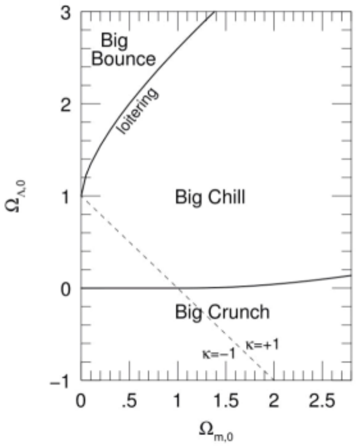
Age of the Univ. in the $\Omega_{m,0} < 1$, $K=0$ model

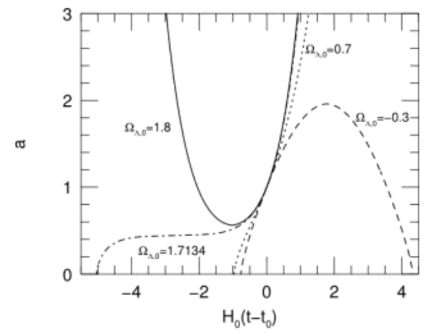
$$t_0 = \frac{2H_0^{-1}}{3(1-\Omega_{m,0})^{1/2}} \sinh^{-1}\left[\sqrt{\frac{1-\Omega_{m,0}}{\Omega_{m,0}}}\right]$$

if $\Omega_{m,0} = 0.3$, $t_0 = 0.964\, H_0^{-1}$

Universes w/ Matter, Curvature & Λ

$$\text{F.E.: } \frac{H^2}{H_0^2} = \frac{\Omega_{m,0}}{a^3} + \frac{(1-\Omega_{m,0}-\Omega_{\Lambda,0})}{a^2} + \Omega_{\Lambda,0}$$

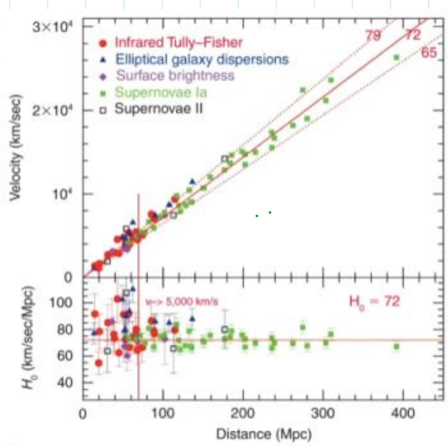




All have $\Omega_{m,0} = 0.3$

Observational Tests

In principle measuring H_0 should be easy.
For small z, the relationship b/w a galaxy's distance d & redshift z is $cz = H_0 d$



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(Wendy L. Freedman, Observatories of the Carnegie Institution of Washington, and NASA)

∴ Measure d & z for a large sample of galaxies and fit a straight line to a plot of cz and d, slope is H_0.

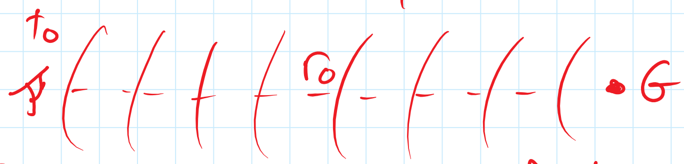
Note: 'peculiar' velocities (ie velocities not related to expansion) cause scatter, so need distant galaxies that sample Hubble flow.

Need to define distance in an expanding Univ.
- the correct way is the proper distance

$$l_{prop} = a(t_0) \int_0^{r_0} \frac{dr}{\sqrt{1-Kr^2}}$$

but no operational way of measuring proper distance (need to either pause the Univ, or have a really fast tape measure).

Consider luminosity distance



From metric, the area of the sphere surrounding G:

$$dA = a^2(t_0)\, r_0^2 \sin\theta \, d\theta \, d\phi$$

$$A = 4\pi a^2(t_0)\, r_0^2$$

Normally, flux received $f = \frac{L}{4\pi d^2}$

You may be tempted to define d_L such that $f = \frac{L}{4\pi r_0^2 a_0^2(t_0)}$ ie $d_L = r_0\, a(t_0)$

This is wrong for 2 reasons.