Monday, September 14, 2015 1:03 PM

This is wrong for 2 reasons:

- units of flux: $\frac{erg}{cm^2 s}$ on Earth
- units of luminosity: $\frac{erg}{s}$ at source

A: photons are redshifted, ie have less energy which gives a reduction in flux by $(1+z)$

B: rate at which photons are received is smaller then when emitted

(ie, if emission rate is $n/\Delta t_e$ and the observed rate is $n/\Delta t_0$, expect reduction in flux of $(1+z)$)

$\therefore$ In an expanding Univ. that has spatial curvature, the relation b/w observed flux f and the luminosity of a distant light source L is

$$f = \frac{L}{4\pi r_0^2 a(t_0)^2 (1+z)^2}$$

Define the luminosity distance to be $d_L = r_0 a(t_0)(1+z)$

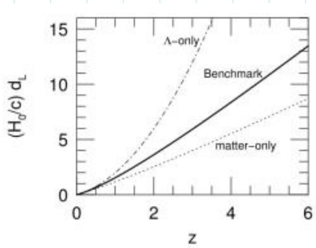
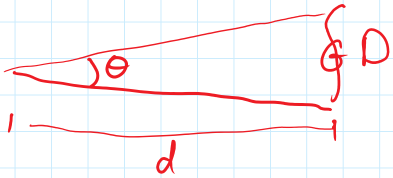


Figure 7.2: The luminosity distance of a standard candle with observed redshift z. The bold solid line gives the result for the Benchmark Model, the dot-dash line for a flat, lambda-only universe, and the dotted line for a flat, matter-only universe.

Angular Diameter Distance

$d\theta$ $\{D$ $\theta = \frac{D}{\ldots} \Rightarrow d = \frac{D}{\ldots}$ if D = standard ruler



$\theta = \dfrac{D}{d} \Rightarrow d = \dfrac{D}{\theta}$ if D = standard ruler

element of circumference: $dC = r_0\, a(t_e)\, \theta$

ie. $D = r_0\, a(t_e)\, \theta$

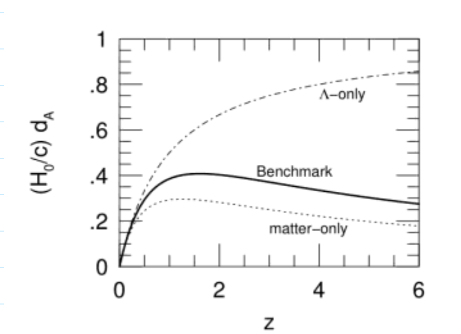
note: $a(t_e)$ <u>not</u> $a(t_0)$

define angular diameter distance

$$d_A = \frac{r_0\, a(t_e)\, \cancel{\theta}}{\cancel{\theta}} = r_0\, a(t_e)\, \frac{a(t_0)}{a(t_0)}$$

recall: $\dfrac{a(t_e)}{a(t_0)} = \dfrac{1}{(1+z)}$

so $d_A = \dfrac{r_0\, a(t_0)}{(1+z)}$ $\left(\text{ie } \dfrac{d_A}{d_L} = \dfrac{1}{(1+z)^2} \right)$



Return to d_{prop} & take $r \ll 1$

$$f(r_0) = \int_0^{r_0} \frac{dr}{\sqrt{1 - Kr^2}} = \int_0^{r_0} dr \left(1 + \tfrac{1}{2}Kr^2 + \cdots\right) \simeq r_0$$

so $d_{prop} \simeq r_0\, a(t_0)$

Return to d_L & d_A and take $z \ll 1$, Then

$$d_L \simeq r_0 a(t_0) \quad \& \quad d_A \simeq r_0 a(t_0)$$

All distance reduce to $r_0 a(t_0)$ for small z

Need large distances to see infl. of cosmology.

Want $r_0 = r_0(z)$ & cosmo.

Consider $K=-1$, $\rho = 0$, no Λ

We have ~~for~~ for propogation of light $(ds)^2 = 0$

$$\int_{t_1}^{t_0} \frac{c\,dt}{a(t)} = \int_0^{r_0} \frac{dr}{\sqrt{1-Kr^2}}$$

$$\dot{a}(t) = \frac{da}{dt} \Rightarrow dt = \frac{da}{\dot{a}}$$

$$\therefore \int_{a_1}^{a_0} \frac{c\,da}{\dot{a}a} = \int_0^{r_0} \frac{dr}{\sqrt{1-Kr^2}}$$

For $K=-1$, $\rho = 0$, no Λ: $\dot{a}^2 = \cancel{\frac{8\pi G \rho_0 a_0^3}{3a}} - Kc^2 = c^2$

so $\dot{a} = c$

$$\int_{a_1}^{a_0} \frac{da}{a} = \int_0^{r_0} \frac{dr}{\sqrt{1+r^2}} = \ln\left(r + (1+r^2)^{1/2}\right)\Big]_0^{r_0}$$

$$= \ln a \Big]_{a_1}^{a_0}$$

$$r_0 + (1+r_0^2)^{1/2} = \frac{a_0}{a_1} = (1+z)$$

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$$r_0 - (1+z) = -(1+r_0^2)^{1/2}$$

$$\cancel{r_0^2} - 2r_0(1+z) + (1+z)^2 = 1 + \cancel{r_0^2}$$

$$\rightarrow r_0 = \frac{(1+z)^2 - 1}{2(1+z)}$$

recall $\dfrac{Kc^2}{a_0^2} = H_0^2(\Omega_0 - 1)$

in this case, $K=-1$, $\Omega_0 = 0$ so $\dfrac{c^2}{a_0^2} = H_0^2$

or $\dfrac{c}{H_0 a_0} = 1$

so $r_0 = \dfrac{c}{H_0 a_0} \dfrac{(1+z)^2 - 1}{2(1+z)}$ $\quad (d_L = r_0 a_0 (1+z), \text{etc})$

Now consider a Univ. w/ matter but no Λ

$$\int_0^{r_0} \frac{dr}{\sqrt{1-Kr^2}} = c\int_{t_1}^{t_0} \frac{dt}{a(t)}$$

but $\dot{a} = \dfrac{da}{dt}$ so $dt = \dfrac{da}{\dot{a}}$

RHS: $c\displaystyle\int_{a_1}^{a_0} \frac{da}{\dot{a}\,a}$ $\qquad$ F.E: $\dfrac{\dot{a}^2}{a^2} = \dfrac{8\pi G\rho}{3} - \dfrac{Kc^2}{a^2}$

Consider $K=+1$

$$a^2\dot{a}^2 = \frac{8\pi G\rho a^4}{3} - c^2 a^2$$

$$= c^2 a\left(\frac{8\pi G\rho a^3}{3c^2} - a\right)$$

matter: $\rho a^3 = \rho_0 a_0^3 = \text{const.}$

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define $a_m = \dfrac{8\pi \rho_0 a_0^3}{3c^2}$

$\therefore$ RHS $= \displaystyle\int_{a_1}^{a_0} \frac{da}{a^{1/2}(a_m - a)^{1/2}}$

What is a_m? $a_m = \left(\dfrac{8\pi G\rho_0}{3H_0^2}\right) \cdot \dfrac{H_0^2 a_0^3}{c^2} = \dfrac{\Omega_{m,0} H_0^2 a_0^3}{c^2}$

$\left(\dfrac{8\pi G\rho_0}{3H_0^2}\right) \to \Omega_{m,0}$

from F.E w/ $K=1$, $\dfrac{c^2}{a_0^2} = H_0^2(\Omega_{m,0} - 1)$

$\rightarrow \dfrac{H_0 a_0^2}{c^2} = \dfrac{1}{\Omega_{m,0} - 1}$

so $a_m = \dfrac{\Omega_{m,0}}{(\Omega_{m,0} - 1)} a_0$

in the integral, substite $a = a_m \sin^2\left(\frac{\theta}{2}\right) = \frac{a_m}{2}(1 - \cos\theta)$

so $da = \frac{a_m}{2} \sin\theta \, d\theta$

denominator in the integral

$a^{1/2}(a_m - a)^{1/2}$ becomes $\frac{a_m}{2}(1 - \cos^2\theta)^{1/2}$

$= \frac{a_m}{2} \sin\theta$

integral becomes $\displaystyle\int_{\theta_1}^{\theta_0} \frac{\cancel{\frac{a_m}{2} \sin\theta} \, d\theta}{\cancel{\frac{a_m}{2} \sin\theta}} = \theta_0 - \theta_1$

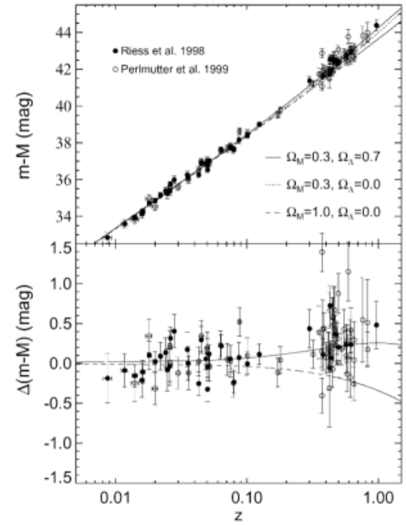

Figure 7.5: Distance modulus versus redshift for type Ia supernovae from the Supernova Cosmology Project (Perlmutter et al.