Phys 4247 - Assignment #5 Solutions

1. a) Solving for time, $\left(\frac{1\text{ sec}}{t}\right) = \left(\frac{T}{2\times10^{10}\text{K}}\right)^2$

$$\therefore t \simeq 1\text{ sec}\left(\frac{T}{2\times10^{10}\text{K}}\right)^{-2}$$

if $T = 3\times10^{25}$K, $\underline{t = 4.4\times10^{-31}\text{ s}}$

b) If $a(t) \propto t$, and $T \propto \frac{1}{a}$ then $T \propto \frac{1}{t}$

Using result from part (a):

$$T = (3\times10^{25}\text{K})\left(\frac{4.4\times10^{-31}\text{s}}{t}\right)$$

When T falls to 3K, $t = \frac{(3\times10^{25}\text{K})}{(3\text{K})} \times (4.4\times10^{-31}\text{s})$

$$\underline{= 4.4\times10^{-6}\text{ s}}$$

2. a) Magnetic monopoles behave as non-relativistic matter:

$$\rho_{mon} = \rho_{mon,GUT}\left(\frac{a_{GUT}}{a}\right)^3 = 10^{-10}\rho_{crit,GUT}\left(\frac{a_{GUT}}{a}\right)^3$$

Assume Universe is radiation dominated and is at the critical density: $\rho_{rad} = \rho_{rad,GUT}\left(\frac{a_{GUT}}{a}\right)^4 = \rho_{crit,GUT}\left(\frac{a_{GUT}}{a}\right)^4$

When these densities are equal:

$$\cancel{\rho_{crit,GUT}}\left(\frac{a_{GUT}}{a}\right)^4 = 10^{-10}\cancel{\rho_{crit,GUT}}\left(\frac{a_{GUT}}{a}\right)^3$$

or $\left(\frac{a_{GUT}}{a}\right)^4 = 10^{-10}\left(\frac{a_{GUT}}{a}\right)^3$

$$\left(\frac{a_{GUT}}{a}\right)^3 = 10^{-10} \rightarrow \frac{a}{a_{GUT}} = 10^{10} \rightarrow \left(\frac{t}{t_{GUT}}\right)^{1/2} = 10^{10}$$

since $a \propto t^{1/2}$ in the rad'n dominated regime.

From the t-T relation given in #1, $\left(\frac{T_{GUT}}{\cancel{2\times10^{10}}K}\right)\left(\frac{\cancel{2\times10^{10}}K}{T}\right) = 10^{10}$

$$\rightarrow T = \frac{T_{GUT}}{10^{10}} = \frac{3\times10^{28}K}{10^{10}} = \underline{3\times10^{18}K}$$

b) Present day: $\frac{\Omega_{mon,0}}{\Omega_{rad,0}} = \frac{\rho_{mon,0}}{\rho_{crit,0}} \Big/ \frac{\rho_{rad,0}}{\rho_{crit,0}} = \frac{\rho_{mon,0}}{\rho_{rad,0}} = \frac{\rho_{mon,GUT}\left(\frac{a_{GUT}}{a_0}\right)^3}{\rho_{rad,GUT}\left(\frac{a_{GUT}}{a_0}\right)^4}$

$$= \frac{10^{-10}\cancel{\rho_{crit,GUT}}\left(\frac{a_{GUT}}{a_0}\right)^3}{\cancel{\rho_{crit,GUT}}\left(\frac{a_{GUT}}{a_0}\right)^4} = 10^{-10}\frac{a_0}{a_{GUT}} = 10^{-10}\left(\frac{t_0}{t_{GUT}}\right)^{1/2}$$

$$= 10^{-10}\left(\frac{T_{GUT}}{T_0}\right) = 10^{-10}\left(\frac{3\times10^{28}K}{3K}\right) = \underline{10^{18}} \;!!$$

Clearly not compatable w/ observations.

3. For the present density of monopoles to equal that of radiation the density of monopoles has to be reduced by ~10^{18} during inflation (according to the situation of Q#2) Since $\rho_{mon} \propto a^{-3}$ that means a has to increase by $\underline{\sim 10^6}$ during inflation.

4. In the development of the ODE for δ, there is no dependence on the overall composition or geometry of the Univ. so the basic equation is still $\ddot{\delta} + 2H\dot{\delta} - \frac{3}{2}\Omega_m H^2 \delta = 0$
Now, consider a negatively curved expanding Univ. w/ $\Omega_m \ll 1$. In this, as the Univ. is almost 'empty', $H = \frac{1}{t}$ (Milne Univ.)

$$\therefore \ddot{\delta} + \frac{2\dot{\delta}}{t} \simeq 0$$

Try a power-law solution: $\delta = Dt^n$

$$n(n-1)Dt^{n-2} + \frac{2Dnt^{n-1}}{t} \simeq 0$$

$$n(n-1) + 2n = 0$$

$$n^2 - n + 2n = n^2 + n = 0$$

$$n(n+1) = 0 \rightarrow n = 0, -1$$

$$\therefore \underline{\delta(t) = D_1 + D_2 t^{-1}}$$

In this Univ, density fluctuations are unable to grow.

5. Need to find radiation and baryonic densities at decoupling
$\Omega_{rad'n,0} = 5\times10^{-5}$, $\Omega_{baryon,0} = 4\times10^{-2}$

$$\therefore \frac{\Omega_{rad'n}}{\Omega_{baryon}} = \frac{5\times10^{-5}}{4\times10^{-2}}\frac{1}{a} = \frac{1.25\times10^{-3}}{a}$$

At decoupling, $z = 1100$, so $a \simeq \frac{1}{1101}$

$$\therefore \frac{\Omega_{rad'n}}{\Omega_{baryon}} \simeq 1.4 \rightarrow \frac{\rho_{rad'n}}{\rho_{baryon}} \simeq 1.4 \rightarrow \frac{\varepsilon_{rad'n}}{\varepsilon_{baryon}} \simeq 1.4$$

$$\therefore \epsilon_{baryon} \simeq \frac{\epsilon_{rad'n}}{1.4} \text{ at decoupling}$$

$$\therefore \epsilon = \epsilon_{rad'n} + \epsilon_{baryon} = \epsilon_{rad'n} + 0.73\,\epsilon_{rad'n} = 1.73\,\epsilon_{rad'n}$$

$$\frac{dP}{d\epsilon} = \frac{dP}{d\epsilon_{rad'n}}\frac{d\epsilon_{rad'n}}{d\epsilon} = \frac{1}{1.73}\frac{dP}{d\epsilon_{rad'n}} = \frac{1}{1.73}\left(\frac{1}{3}\right)$$

$$\therefore C_s = c\sqrt{\frac{dP}{d\epsilon}} = \frac{1}{\sqrt{1.73}}\frac{c}{\sqrt{3}} = \underline{0.76\left(\frac{c}{\sqrt{3}}\right)}$$

Since the Jean's Length $\propto C_s$, the contribution from the baryons to the Jean's Length at decoupling only decreases it by $\underline{\sim 24\%}$ conpared to the estimate using just radiation.