

Lecture 1

Part 1: Radiation Processes

Key ref: Rybicki & Lightman (1979)

Brief review of classical E&M. Work in Gaussian/cgs units to be compatible w/ most books. Thus, in vacuum

$\epsilon_0 = \mu_0 = 1$; \vec{E} ; \vec{B} have same units (Gauss)

M.E. in vacuum

$$\vec{\nabla} \cdot \vec{E} = 0$$

$$\vec{\nabla} \cdot \vec{B} = 0$$

$$\vec{\nabla} \times \vec{E} = -\frac{1}{c} \frac{\partial \vec{B}}{\partial t}$$

$$\vec{\nabla} \times \vec{B} = \frac{1}{c} \frac{\partial \vec{E}}{\partial t}$$

From these we get the wave equations:

$$\nabla^2 \vec{E} - \frac{1}{c^2} \frac{\partial^2 \vec{E}}{\partial t^2} = 0 ; \quad \nabla^2 \vec{B} - \frac{1}{c^2} \frac{\partial^2 \vec{B}}{\partial t^2} = 0$$

With plane wave solutions:

$$\vec{E}(\vec{r}, t) = \vec{E}_0 e^{i(\omega t - \vec{k} \cdot \vec{r})}$$

$$; \quad \vec{B}(\vec{r}, t) = \vec{B}_0 e^{i(\omega t - \vec{k} \cdot \vec{r})}$$

\vec{k} is the wave vector: $|\vec{k}| = \frac{2\pi}{\lambda}$; $\omega = 2\pi\nu$
 and points in the direction of λ and $\lambda\nu = c$

propagation...

M.E. tells us that \vec{E}_0 ; \vec{B}_0 ; \vec{k} are all mutually perpendicular and $|\vec{E}_0| = |\vec{B}_0|$

The electric & magnetic fields contain energy w/ densities $U_E = \frac{E^2}{8\pi}$; $U_B = \frac{B^2}{8\pi}$. Thus propagating \vec{E} ; \vec{B} fields in radiation carry energy with a flux [energy/area/time] given by the Poynting vector

$$\vec{S} = \frac{c}{4\pi} (\vec{E} \times \vec{B}).$$

$$\text{For plane waves, } \langle S \rangle = \frac{c}{4\pi} \langle |\vec{E} \times \vec{B}| \rangle = \frac{c}{4\pi} \langle E^2 \rangle \\ = 2c \langle U_E \rangle = c (\langle U_E \rangle + \langle U_B \rangle)$$

where U_{rad} is the energy density $\stackrel{=c}{=} U_{\text{rad}}$ the rad'n field.

(Note that this is flux = (speed of object) (density of object))

Power Spectra

The detected/emitted radiation will be a function of time, which, in general, can take any shape
 $\uparrow |\vec{E}_0|$



One cannot characterize this radiation w/ a wave equation at a precise instant of time, knowing only \vec{E}_0 at one point.

If we measure $|\vec{E}_0|(t)$ for Δt , we still can only define the wave train to within a freq. resolution $\Delta\omega$ where $\Delta\omega\Delta t > 1$

Back to our pulse, how much energy does it contain

At one point in time: $\langle S \rangle = \frac{dW}{dt dA} = \frac{c^2}{4\pi} E^2(t)$

energy

So, the total energy per unit area in the pulse is

$$\frac{dW}{dA} = \frac{c}{4\pi} \int_{-\infty}^{\infty} E^2(t) dt$$