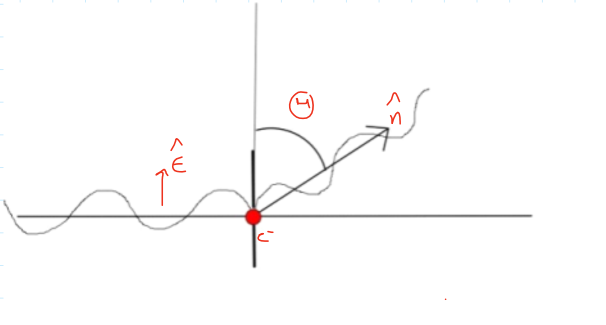


What is interesting to know is what fraction of the incident wave is re-emitted in a particular direction.

The incident flux on the e^- is $\langle S \rangle = \frac{c}{8\pi} E_0^2$

Define the differential cross-section $d\sigma$ for scattering into $d\Omega$ so that $\frac{dP}{d\Omega} = \langle S \rangle \frac{d\sigma}{d\Omega} = \frac{cE_0^2}{8\pi} \frac{d\sigma}{d\Omega}$

$$\therefore \left(\frac{d\sigma}{d\Omega}\right)_{pol.} = \frac{e^4}{m^2c^4} \sin^2 \Theta$$

And the total cross-section σ is

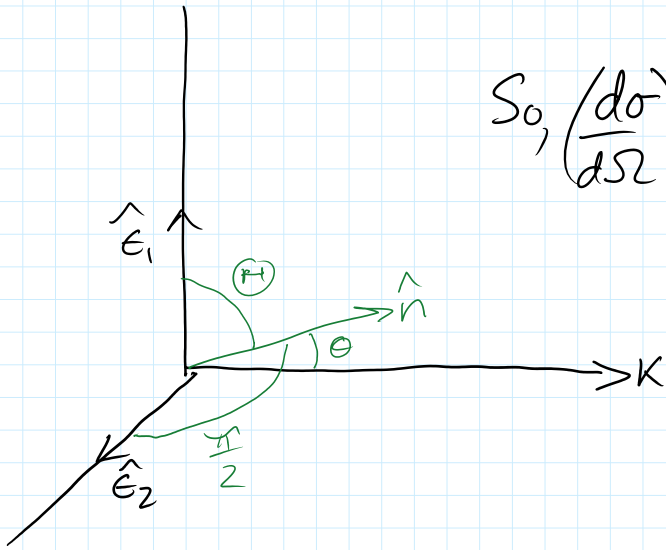
$$\sigma = \int \frac{d\sigma}{d\Omega} d\Omega = \frac{2\pi e^4}{m^2c^4} \int_0^\pi \sin^3 \Theta \, d\Theta = \frac{8\pi e^4}{3m^2c^4} \quad \text{(freq. indep.)}$$

For an e^-, $\sigma = \sigma_T$ = Thomson cross-section $= 6.65 \times 10^{-25}\ \text{cm}^2$

Valid for $h\nu \ll mc^2$ & when e^- is moving non-relativistically.

(More generally, the e^- will recoil & gain energy. And when $h\nu \sim mc^2$, the cross-section requires QM corrections)

For an unpolarized beam, we can regard it as the superposition of 2 independent orthogonal polarized beams



So, $$\left(\frac{d\sigma}{d\Omega}\right)_{unpol.} = \frac{1}{2}\left[\left(\frac{d\sigma}{d\Omega}(\Theta)\right)_{pol} + \left(\frac{d\sigma}{d\Omega}(\pi/2)\right)_{pol.}\right]$$

$$= \frac{1}{2}\left[\frac{e^4 \sin^2\Theta}{m^2c^4} + \frac{e^4}{m^2c^4}\right]$$

$$= \frac{e^4}{2m^2c^4}\left(1 + \sin^2\Theta\right)$$

$$= \frac{e^4}{2m^2c^4}\left(1 + \cos^2\theta\right)$$

ie, only depends on angle
b/w incident & scattered directions

Note: 1) $\sigma_{unpol.} = \int \left(\frac{d\sigma}{d\Omega}\right)_{unpol.} d\Omega = \sigma_{pol.}$ (verify!)

makes sense since e^- at rest has no preferred direction

2) $\left(\frac{d\sigma}{d\Omega}\right)_{unpol.}$ is the same if $\theta \to -\theta$
(forward/backward symmetry)

3) The scattered wave is polarized w/ fraction
$$\Pi = \frac{1-\cos^2\theta}{1+\cos^2\theta}$$

Define the Thomson optical depth along a path,

$$\tau_T = \int ds\, n_e \sigma_T \quad \text{where } n_e = \text{free } e^- \text{ \# density}$$

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When $\tau_T \ll 1$, it measures the probability that a photon will be scattered. A source w/ $\tau_T \gg 1$ is said to be optically thick to Thomson scattering. Photons escape by diffusing out w/ a mean speed $\sim \frac{c}{\tau_T}$.