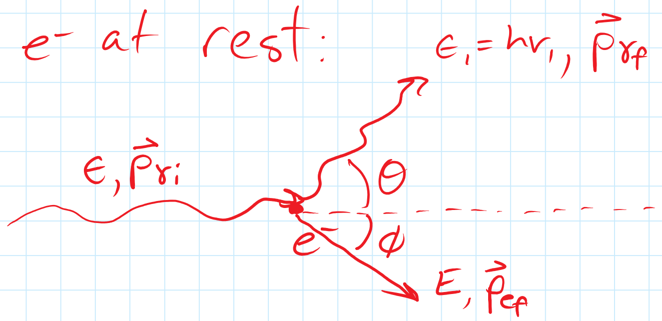
Compton Scattering & Comptonization

When the incident radiation has energy $\sim mc^2$ quantum effects become important and the scattering is best described w/ the scattering of photons. QM effects appear in 2 ways. First, through the kinematics of the scattering process b/c the photon posses a momentum $\frac{h\nu}{c}$ as well as $\epsilon = h\nu$.

Consider an e^- at rest:



Conservation of energy & momentum gives the solution for the scattered energy

$$\epsilon_1 = \frac{\epsilon}{1 + \left(\frac{\epsilon}{mc^2}\right)(1-\cos\theta)} \leq \epsilon$$

$\therefore$ The photon is always redshifted b/c the recoil of the e^- carries away some of the incident photon's energy. Note that if $\epsilon \ll mc^2$ we recover Thomson scattering.

The 2nd quantum effect is to change the cross-section - it is now energy dependent. The differential cross-section for unpol. radiation is shown in QED to be

$$\frac{d\sigma}{d\Omega} = \frac{e^4}{2m^2c^4}\left(\frac{\epsilon}{\epsilon_1} + \frac{\epsilon_1}{\epsilon} - \sin^2\theta\right)\frac{\epsilon_1^2}{\epsilon^2} \quad \propto \frac{1}{\epsilon} \text{ for large } \epsilon$$

(Check that it reduces to classical $\frac{d\sigma}{d\Omega}$ when $\epsilon_1 \simeq \epsilon$)

The principle effect is to reduce the cross-section from its classical value as ϵ becomes large. Thus, Compton scattering becomes less efficient at large ϵ.

The total cross-section is (Klein-Nishina cross-section)

$$\sigma = \sigma_T \frac{3}{4}\left[\frac{1+x}{x^3}\left\{\frac{2x(1+x)}{1+2x} - \ln(1+2x)\right\} + \frac{1}{2x}\ln(1+2x) - \frac{(1+3x)}{(1+2x)^2}\right] \quad \left(x = \frac{h\nu}{mc^2}\right)$$

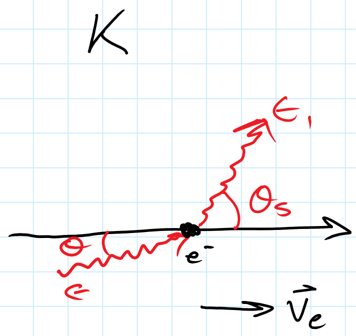
when $x \ll 1$, $\sigma \simeq \sigma_T\left(1 - 2x + \frac{26}{5}x^2 + \cdots\right)$

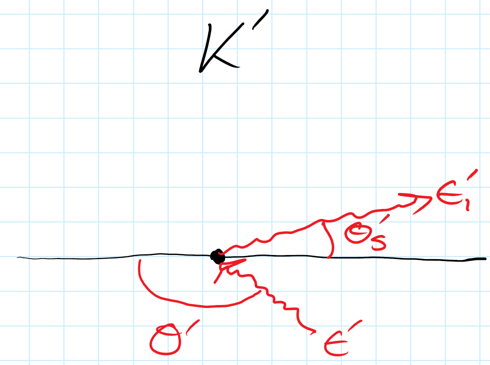
$x \gg 1$, $\sigma = \frac{3}{8}\frac{\sigma_T}{x}\left[\frac{1}{2} + \ln 2x\right]$

- If the e^- is moving then, if it has sufficient kinetic energy compared to the incoming photon, net energy may be transferred from the e^- to the photon. Thus, the scattered photon gains energy. This is called inverse Compton scattering or Compton up-scattering.

Easiest to see for highly-relativistic e^- w/ Lorentz factor γ

Consider the geometry in the observer's frame (K) and e^- rest frame (K')





We want ϵ_1 in terms of ϵ. Make 3 transformations

$$\epsilon \xrightarrow{\text{Doppler}} \epsilon' \xrightarrow{\text{Compton}} \epsilon_1' \xrightarrow{\text{Doppler}} \epsilon_1$$

The Doppler shift formulas are $\epsilon' = \epsilon\gamma(1-\beta\cos\theta)$

$$\epsilon_1 = \epsilon_1'\gamma(1+\beta\cos\theta_s')$$

The Compton formula after averaging over θ is

$$\epsilon_1' = \frac{\epsilon'}{1+\left(\frac{\epsilon'}{mc^2}\right)}$$

$$\therefore \quad \epsilon_1 = \gamma^2\epsilon\left(1+\frac{\gamma\epsilon}{mc^2}\right)^{-1}(1-\beta\cos\theta)(1+\beta\cos\theta_s')$$

$$\simeq \gamma^2\epsilon\Big(1+\underbrace{\frac{\gamma\epsilon}{mc^2}}_{\text{recoil term; small if } \gamma\epsilon \ll mc^2}\Big)^{-1}$$ since θ & θ_s' are characteristically $\sim\frac{\pi}{2}$

The process can convert a low-energy photon to a high-energy one by a factor $\sim\gamma^2$. (Note: for specific angles, the scattered energy can be less than the initial)