e.g. $\gamma = 10^3$, $\nu = 10^{10}$ Hz $\rightarrow \nu_s = \nu\gamma^2 \sim 10^{16}$ Hz

(ie, hard UV photon $\rightarrow$ soft X-rays)

Note that if $\gamma\epsilon \sim 100$ keV the recoil effect & KN reductions are small so $\gamma^2\epsilon$ can be very large.

The max. possible energy is $\epsilon_1 = \gamma mc^2 + \epsilon$ or $\sim \gamma mc^2$ as γ becomes large.

Condition for energy to flow from the e^- to the photon:

$$\epsilon_1 > \epsilon, \quad \text{or} \quad \gamma^2 > 1 + \frac{\gamma\epsilon}{mc^2}$$

$$(\gamma^2 - 1) > \frac{\gamma\epsilon}{mc^2}$$

$$(\gamma-1)(\gamma+1)\,mc^2 > \gamma\epsilon$$

$$(\gamma-1)\,mc^2 > \frac{\gamma\epsilon}{\gamma+1}$$

$$\text{KE of } e^- > \left(\frac{\gamma}{\gamma+1}\right)\epsilon \sim (0.5 \rightarrow 1)\,\epsilon$$

Inverse Compton Power for Relativistic e^- Moving Through an Isotropic Rad'n Field

In the particle's rest frame, the scattered energy emission rate is

$$\frac{dE'}{dt'} = c\sigma_T \int \epsilon' \underbrace{\left(\frac{dn}{d\epsilon'}\right)}_{\#\text{ density of incident photons}} d\epsilon'$$

Assume Thomson scattering in rest-frame, ie $\epsilon_1' = \epsilon'$ (also implies no KN x-section.). Also, $\frac{dE}{dt} = \frac{dE'}{dt'}$

$$\therefore \frac{dE}{dt} = c\sigma_T \int \epsilon'^2 \left(\frac{dn}{d\epsilon'}\frac{1}{\epsilon'}d\epsilon'\right) = c\sigma_T \int \epsilon'^2 \frac{dn}{d\epsilon}\frac{1}{\epsilon}d\epsilon$$

↳ Lorentz invariant

Doppler shift ϵ' back to lab frame: $\epsilon' = \epsilon\,\gamma(1-\beta\cos\theta)$

$$\therefore \frac{dE}{dt} = c\sigma_T \gamma^2 \int (1-\beta\cos\theta)^2 \epsilon \frac{dn}{d\epsilon} d\epsilon$$

For an isotropic dist'n of photons $\langle (1-\beta\cos\theta)^2 \rangle = 1 + \frac{1}{3}\beta^2$ since $\langle \cos\theta \rangle = 0$; $\langle \cos^2\theta \rangle = \frac{1}{3}$

$$\therefore \frac{dE}{dt} = c\sigma_T \gamma^2 (1 + \tfrac{1}{3}\beta^2) U_{ph}$$

where $U_{ph} \equiv \int \epsilon \frac{dn}{d\epsilon} d\epsilon$ is the initial photon energy density

This is the energy gained by the photon field due to the scattering of the low-energy photons. To find the energy loss rate of the e^-, we must subtract out the rate of change in the initial photon field $\frac{dE_{ph}}{dt} = \sigma_T c U_{ph}$

So, the net power lost by the e^- and converted into increased radiation is

$$\frac{dE_{rad}}{dt} = c\sigma_T U_{ph}\left[\gamma^2(1+\tfrac{1}{3}\beta^2) - 1\right]$$

but $\gamma^2 = \frac{1}{1-\beta^2}$ or $\gamma^2 - \gamma^2\beta^2 = 1$ or $\gamma^2 - 1 = \gamma^2\beta^2$

then $\frac{dE_{rad}}{dt} = c\sigma_T U_{ph}\left(\gamma^2 - 1 + \frac{\gamma^2\beta^2}{3}\right) = c\sigma_T U_{ph}\left(\gamma^2\beta^2 + \frac{\gamma^2\beta^2}{3}\right)$

or $\boxed{P_{comp} = \frac{4}{3}\sigma_T c \gamma^2 \beta^2 U_{ph}}$ $\quad (\gamma h\nu \ll mc^2)$

[if recoil is incl. in e^- r.f.

$$P = \frac{4}{3}\sigma_T c \gamma^2 \beta^2 U_{ph}\left[1 - \frac{63}{10}\frac{\gamma\langle\epsilon^2\rangle}{mc^2\langle\epsilon\rangle}\right]$$

where $\langle\epsilon^2\rangle$; $\langle\epsilon\rangle$ are mean values integrated over U_{ph}. In this case, energy can be given to e^-]

Recall that the synchrotron power emitted by an e^- is $P_{syn} = \frac{4}{3}\sigma_T c \gamma^2 \beta^2 U_B$

so $\frac{P_{sync}}{P_{comp}} = \frac{U_B}{U_{ph}}$ for any e^- velocity

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Comptonization

At photon energies > 10 keV, the total photoelectric x-section of an astrophysical plasma can be small compared to the Compton crosssection. ∴ The opacity can be dominated by Compton scattering. In fact, if the plasma is mostly ionized then Compton scattering is the dominant photon-matter interaction, even for lower-energy photons.