Deborah - Kilonovas

Chris - Magnetically driven jets

Pranav - Magnetars

To determine the broadband spectral properties for both saturated and unsaturated Comptonization, use the Komptoneets equation, which describes the evolution of the photon distribution due to repeated, non-relativistic scatterings

$$\frac{dn}{dt_c} = \left(\frac{KT}{mc^2}\right)\frac{1}{x}\frac{\partial}{\partial x}\left[x^4\left(\frac{\partial n}{\partial x} + n + n^2\right)\right]$$

where $t_c \equiv (n_e \sigma_T c)t$ is the Compton time, the mean time b/w scattering

$$x = \frac{\hbar\omega}{KT}$$

n = photon phase space density, ie. a Bose-Einstein distribution and is conserved

This equation is basically a diffusion equation in freq. space.
1st term: Doppler heating
2nd term: Compton recoil
3rd term: induced scattering (QM effect; important when $n \gg 1$)

In the astrophysical context, want steady-state solutions due to a source of soft photons

Define $Q(x) = \#$ source photons produce per t_c per state

$= Q_0(x) \quad x < x_s < 1 \;;\; Q_0 = 0$ for $x > x_s$

Assume that the escape prob./scatter is $\frac{1}{N} = \frac{1}{\max(\tau, \tau^2)}$

and neglect stimulated scattering ($n \ll 1$)

Then the modified Kompotoneets eqn for this situation is

$$\frac{\partial n}{\partial t_c} = \frac{kT}{mc^2}\frac{1}{x^2}\frac{\partial}{\partial x}\left[x^4\left(\frac{\partial n}{\partial x} + n\right)\right] + Q(x) - \frac{n}{\max(\tau,\tau^2)}$$

$= 0$ for steady state

First, what is the spectrum for $x \gg x_s$ (here, $Q = 0$)

$$\max(\tau,\tau^2)\frac{kT}{mc^2}\frac{\partial}{\partial x}\left[x^4\left(\frac{\partial n}{\partial x} + n\right)\right] - x^2 n = 0$$

or

$$\frac{1}{4}\,y\,\frac{kT}{mc^2}\frac{\partial}{\partial x}\left[x^4\left(\frac{\partial n}{\partial x}\right) + n\right)\Big] - x^2 n = 0$$

For $x \gg 1$, can neglect x^2 term, so

$$\frac{y}{4}\frac{kT}{mc^2}\frac{\partial}{\partial x}\left[x^4\left(\frac{\partial n}{\partial x}\right) + n\right] = 0$$

$$\Rightarrow x^4\left(\frac{\partial n}{\partial x} + n\right) = \text{const.}$$

$$\rightarrow \frac{\partial n}{\partial x} = -n + \frac{\text{const}}{x^4}$$

As x gets big, $n \simeq e^{-x} = e^{-h\nu/kT}$

Since intensity $I \propto \nu^3 n$, the spectrum has a Wien shape

Second, $x \gg x_s$ (so $Q=0$) but $x \ll 1$. Here we can neglect n compared to $\frac{\partial n}{\partial x}$

then $\frac{1}{4} y \frac{kT}{mc^2} \frac{\partial}{\partial x} x^4 \left(\frac{\partial n}{\partial x}\right) - x^2 n = 0$

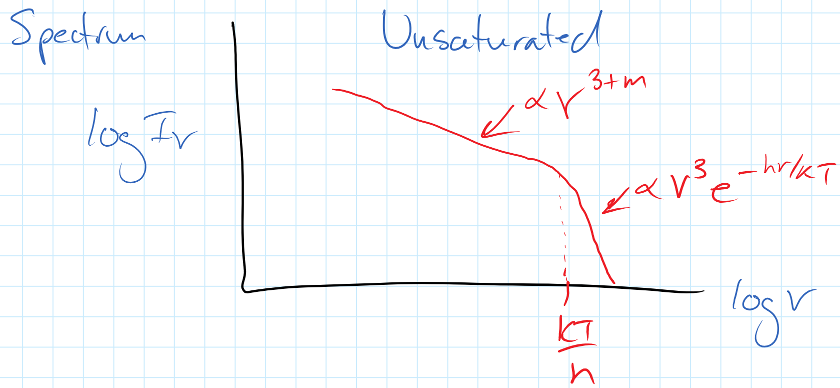
Trying a power-law of the form $n \propto x^m$ we find

$$m = -\frac{3}{2} \pm \left(\frac{9}{4} + \frac{4}{y}\right)^{1/2}$$

Thus, unsaturated Comptonization can give a power-law continuum from a thermal process.

If $y \ll 1$, take the negative root

$y \gg 1$, then Comptonization is saturated & get the low-ν limit of the Wien spectrum



If measure slope & cutoff can observationally determine kT & Y.

Compton Heating & Cooling

Recall net energy exchanged in a collision

$$\left\langle \frac{\Delta \epsilon}{\epsilon} \right\rangle = \frac{4kT - \epsilon}{mc^2}$$

Let $N_\nu d\nu$ be the # of photon per unit volume w/ ν b/w $[\nu, \nu + d\nu]$. Then a change in the radiation energy density in that freq. interval is

$$\Delta \left(N_\nu \epsilon \, d\nu \right) = N_\nu \frac{\epsilon}{mc^2} (4kT - \epsilon) \, d\nu$$

where $\epsilon = h\nu$

Then $\frac{\partial U}{\partial t}$ is found by x rhs by the rate of scatterings $n_e c \sigma_T$ & integrating over all freq:

$$\frac{\partial U}{\partial t} = \frac{n_e \sigma}{mc} \int_0^\infty d\nu \, U_\nu(\nu) \left(4kT - h\nu \right)$$