For a BH w/ no hard surface, the accretion energy does not have to all come out as rad'n - some can just fall into the BH & add to its mass. Minimum radius of BH accretion (its 'surface') is Schwarzschild radius $r_s = \frac{2GM}{c^2}$. Radiation is released w/ an efficiency η

$$L_{acc} = \eta \dot{M} c^2$$

We will see that the most radiative efficient accretion occurs in a thin disk. GR gives a min value to the innermost stable circular orbit (ISCO).

$\therefore$ The max. amount of energy that can be released via BH accretion is for a gas particle falling from ∞ through the disk to the ISCO.

$$\Rightarrow \eta_{max} = \frac{(\text{max. available grav. p.e.})}{(\text{rest mass energy})} = \frac{(\text{max binding energy})}{(\text{rest mass energy})}$$

For a Schwarzschild BH, the ISCO is at $r_{min} = 3R_s = \frac{6GM}{c^2}$

A Newtonian calculation gives

$$\eta_{max} = \frac{(GMm/2r_{min})}{mc^2} = \frac{(GMmc^2/12GM)}{mc^2} = \frac{1}{12} = 0.08\dot{3}$$

The factor of 2 comes from taking into account the orbital KE of the particle.

A proper relativistic calculation there is a reduction in the max. binding energy ; find $\eta_{max} = 0.058$

For a Kerr BH, the max efficiency depends on the value of $a = \frac{J}{M}$ for the BH. For a maximal rotating BH w/ $a=1$, $r_{min} \sim \frac{GM}{c^2}$; $\eta_{max} = 0.423$ for prograde rotation.

[Thorne 1974 ; $a_{max} = 0.998$; $\eta_{max} = 0.30$ (b/c of photons w/ neg. ang. mom.)]

For a counter-rotating BH, $r_{min} \sim 9\,GM/c^2$; $\eta_{max} \simeq 0.03$.

An observational constraint on efficiency (Fabian 1979) that, for spherical sources, is model independent. Suppose the luminosity L is radiated by a variable source over a timescale Δt. If the source has a significant e^- scattering optical depth $\tau \gtrsim 1$, the observed timescale of variation must satisfy $\Delta t \gtrsim (1+\tau)\frac{R}{c}$ where the effective diffusion velocity of the rad'n in the source is $\frac{c}{\tau}$. If the density in the source is n the mass involved is $M = \frac{4}{3}\pi R^3 n m_p$; the luminosity is $L = \frac{\eta M c^2}{\Delta t}$. So, since $\tau = n\sigma_T R$,

$$\eta = \frac{\Delta t\, L}{Mc^2} = \frac{\Delta t\, L}{\frac{4}{3}\pi R^3 n m_p c^2} = \frac{\Delta t\, L\, \sigma_T}{\frac{4}{3}\pi R^2 \tau m_p c^2}$$

but $\Delta t\, c \gtrsim (1+\tau) R$

or $(\Delta t c)^2 \geq (1+\Upsilon)^2 R^2$

$$\eta \simeq \frac{3\cancel{(\Delta t)}\, L \sigma_T (1+\Upsilon)^2}{4\pi (\Delta t)^2 c^4 \Upsilon m_p}$$

Since $\Upsilon > 1$, $\frac{(1+\Upsilon)^2}{\Upsilon} \simeq \Upsilon$

and remembering this $\Upsilon > 1$

$$\eta \geq \frac{3 L \sigma_T}{4\pi (\Delta t) c^4 m_p} \Rightarrow \eta > \frac{L}{(2 \times 10^{42} \frac{\text{ergs}}{\text{s}}) \Delta t}$$

This argument will yield an estimate of η even for a general source of opacity.