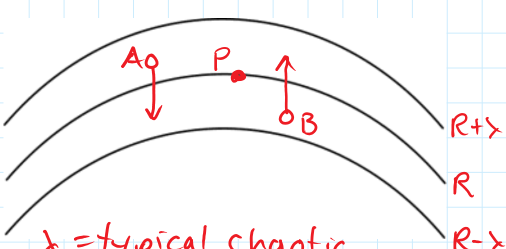
Consider the radius at which the thermal energy per unit mass $\simeq$ grav. p.e. per unit mass. At this point, gas will start to slowly accrete to the center

$$\frac{1}{2} c_s^2(\infty) \simeq \frac{GM}{r}$$

then $$r_{acc} \simeq \frac{2GM}{c_s^2(\infty)}$$

Analytical Thin Accretion Disk Theory

Most gas possesses some ang. mom. and will not plunge right into a BH or a NS. Collisions will cancel out the vertical motions but ang. mom. will cause the gas to settle into a disk w/ angular speeds approx. Keplerian: $\Omega_K(R) = \left(\frac{GM}{R^3}\right)^{1/2}$



λ = typical chaotic length scale

But because gas particles will jiggle about, viscous stresses are generated which transport ang. mom. orthogonal to the gas motion.

Gas is flowing in ϕ direction

Gas elements A & B are constantly exchanged across the surface w/ speed $\sim \tilde{v}$ and travel a distance $\sim \lambda$ before interacting

No net transfer of matter, so the mass flux in both directions at P is Mass flux $\simeq \rho \tilde{v}$ $\qquad \rho$ = gas density

$$\text{Mass rate} \simeq \rho \check{v} A = \rho \check{v} (H\ell)$$

where H is the scale heigh + ℓ is the arc-lenght along surface

So $$\frac{\text{Mass rate}}{\text{arc length}} \simeq \rho \hat{v} H$$

But the two masses carry diff. ang. momenta $\therefore$ there will be a viscous torque exerted on the outer stream by the inner stream

Imagine an observer at P rotating w/ $\Omega(R)$, the fluid at $(R-\frac{\lambda}{2})$ will appear to move w/ velocity

$$\left(R-\tfrac{\lambda}{2}\right)\Omega\left(R-\tfrac{\lambda}{2}\right) - \Omega(R)R$$

$\therefore$ The average rate of ang. mom./arc length through R in the outward direction is

$$\underbrace{H\rho\check{v}}_{\frac{\text{mass rate}}{\text{arc length}}} \underbrace{\left(R-\tfrac{\lambda}{2}\right)}_{\text{'r'}} \underbrace{\left[\left(R-\tfrac{\lambda}{2}\right)\Omega\left(R-\tfrac{\lambda}{2}\right) - \Omega(R)R\right]}_{\text{'v'}}$$

inward direction:

$$H\rho\check{v}\left(R+\tfrac{\lambda}{2}\right)\left[\left(R+\tfrac{\lambda}{2}\right)\Omega\left(R+\tfrac{\lambda}{2}\right) - \Omega(R)R\right]$$