There can be 3 regimes in an accretion disk:

1) inner disk $\left(\kappa = \frac{\sigma_T}{m_p} = 0.4\ cm^2 g^{-1};\ P_{rad} > P_{gas}\right)$

2) middle disk $\left(\kappa = \frac{\sigma_T}{m_p} = 0.4\ cm^2 g^{-1};\ P_{gas} > P_{rad}\right)$

3) outer disk $\left(\kappa = \text{Kramer's Law} = 6.6 \times 10^{22} \rho T^{-7/2}\ cm^2/g\right.$

$\left. P_{gas} > P_{rad} \right)$

approx. expression which combines bf & ff opacity for solar abundances

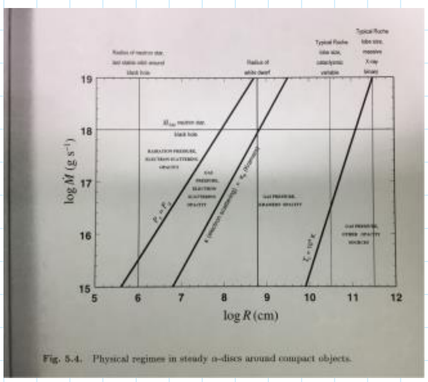

Fig. 5.4. Physical regimes in steady α-discs around compact objects.

Solution for outer disk. Gas pressure dominated w/ Kramer's opacity. Write $\left(1 - \left(\frac{R_*}{R}\right)^{1/2}\right) = y^4$ & write D for the RHS of the energy eqn: $\frac{4\sigma}{3\Upsilon} T_c^4 = D(R)$

but $\Upsilon = \Sigma\kappa = \Sigma(6.6\times10^{22})\rho T_c^{-7/2}$

$$\therefore T_c^{15/2} = \frac{3(6.6\times10^{22})}{4\sigma} \rho \Sigma D(R)$$

but $\rho = \frac{\Sigma}{H}$ and $H = \frac{c_s R^{3/2}}{(GM)^{1/2}}$ so $\rho = \frac{\Sigma (GM)^{1/2}}{c_s R^{3/2}}$

$$\therefore T_c^{15/2} = \frac{3(6.6\times10^{22})}{4\sigma} \frac{\Sigma^2 (GM)^{1/2}}{c_s R^{3/2}} D(R)$$

$$\frac{}{4\sigma} \quad \overline{c_s R^{3/2}}$$

but $c_s^2 = \frac{P}{\rho} = \frac{\cancel{\rho} k T_c}{\mu m_p \cancel{\rho}} \rightarrow c_s = \sqrt{\frac{k}{\mu m_p}}\, T_c^{1/2}$

$$\therefore\ T_c^8 = \frac{3(6.6\times10^{22})}{4\sigma} G^{1/2} \sqrt{\frac{\mu m_p}{k}}\, \Sigma^2 M^{1/2} D(R) R^{-3/2}$$

Combine this w/ 7 & 8

$7 \rightarrow \Sigma = \frac{\dot{M} y^4}{3\pi\nu}$; $8 \rightarrow \nu = \alpha c_s H = \alpha \frac{kT_c}{\mu m_p} \frac{R^{3/2}}{(GM)^{1/2}}$

$$\rightarrow \Sigma = \frac{\dot{M} \mu m_p (GM)^{1/2} y^4}{\alpha k T_c R^{3/2} 3\pi}$$

$$\Sigma^8 = \left(\frac{\mu m_p}{k}\right)^8 G^4 \left(\frac{1}{3\pi}\right)^8 y^{32} \dot{M}^8 M^4 \alpha^{-8} T_c^{-8} R^{-12}$$

Sub in for T_c^{-8} from above

$$\Sigma^{10} = \left(\frac{\mu m_p}{k}\right)^{15/2} G^{7/2} \left(\frac{1}{3\pi}\right)^8 \left(\frac{3(6.6\times10^{22})}{4\sigma}\right)^{-1} \dot{M}^8 M^{7/2} \alpha^{-8} D^{-1} R^{-21/2} y^{3/2}$$

Sub in for $D(R)$ & ASA

$$\Sigma = (5.358\times10^{-12})\, \dot{M}^{7/10} M^{1/4} \alpha^{-4/5} R^{-3/4} y^{14/5}$$

e.g. a NS scaling would be $\dot{M} = 10^{16}\,\frac{g}{s}$, $M = M_\odot$ & $R = 10^{10}$ cm

$$\Sigma = 5.67 \left(\frac{M}{M_\odot}\right)^{1/4} \left(\frac{\dot{M}}{10^{16}\,\frac{g}{s}}\right)^{7/10} \alpha^{-4/5} \left(\frac{R}{10^{10}\,\text{cm}}\right)^{-3/4} y^{14/5} \quad \frac{g}{cm^2}$$

a BH scaling gives

$$\Sigma = (4.487\times10^5) \left(\frac{M}{M_\odot}\right)^{1/5} \left(\frac{\eta}{0.1}\right)^{-7/10} \alpha^{-4/5} \left(\frac{\dot{M}}{\dot{M}_{Edd}}\right)^{7/10} \left(\frac{R}{R_{sch}}\right)^{-3/4} y^{14/5} \quad \frac{g}{cm^2}$$

The total sol'n for outer disk is: (w/ BH scaling)

$$H = (2144)\, \alpha^{-1/10} \left(\frac{M}{\ }\right)^{9/10} \left(\frac{R}{\ }\right)^{9/8} \left(\frac{\dot{M}}{\ }\right)^{-3/20} \left(\frac{\eta}{\ }\right)^{-3/20} y^{3/5} \quad \text{cm}$$

$$H = (2144)\,\alpha^{-1/10}\left(\frac{M}{M_\odot}\right)^{9/10}\left(\frac{R}{R_{sch}}\right)^{9/8}\left(\frac{\dot M}{\dot M_{Edd}}\right)^{3/20}\left(\frac{\eta}{0.1}\right)^{-3/20} y^{3/5} \ \text{cm}$$

$$\rho = (209.7)\,\alpha^{-7/10}\left(\frac{M}{M_\odot}\right)^{-7/10}\left(\frac{\eta}{0.1}\right)^{-11/20}\left(\frac{\dot M}{\dot M_{Edd}}\right)^{11/20}\left(\frac{R}{R_{sch}}\right)^{-15/8} y^{11/5} \ \frac{\text{g}}{\text{cm}^3}$$

$$T_c = (1.76\times10^8)\,\alpha^{-1/5}\left(\frac{\eta}{0.1}\right)^{-3/10}\left(\frac{M}{M_\odot}\right)^{-1/5}\left(\frac{\dot M}{\dot M_{Edd}}\right)^{3/10}\left(\frac{R}{R_{sch}}\right)^{-3/4} y^{6/5} \ \text{K}$$

$$\Upsilon = (88.2)\,\alpha^{-4/5}\left(\frac{M}{M_\odot}\right)^{1/5}\left(\frac{\eta}{0.1}\right)^{-1/5}\left(\frac{\dot M}{\dot M_{Edd}}\right)^{1/5} y^{4/5}$$

$$\nu = (3.30\times10^{11})\,\alpha^{4/5}\left(\frac{M}{M_\odot}\right)^{4/5}\left(\frac{\eta}{0.1}\right)^{-3/10}\left(\frac{\dot M}{\dot M_{Edd}}\right)^{3/10}\left(\frac{R}{R_{sch}}\right)^{3/4} y^{6/5} \ \frac{\text{cm}^2}{\text{s}}$$

$$V_R = (1.68\times10^6)\,\alpha^{4/5}\left(\frac{\eta}{0.1}\right)^{-3/10}\left(\frac{M}{M_\odot}\right)^{-1/5}\left(\frac{\dot M}{\dot M_{Edd}}\right)^{3/10}\left(\frac{R}{R_{sch}}\right)^{-1/4} y^{-14/5} \ \frac{\text{cm}}{\text{s}}$$

Note that α doesn't enter into the solution w/ a high power

$$\left(\frac{H}{R}\right) = (0.0073)\left(\frac{M}{M_\odot}\right)^{-1/10}\left(\frac{R}{R_{sch}}\right)^{1/8}\left(\frac{\eta}{0.1}\right)^{-3/20}\left(\frac{\dot M}{\dot M_{Edd}}\right)^{3/20}\alpha^{-1/10} y^{3/5} \ll 1$$

Check where the assumption of Kramer's opacity breaks down. $\kappa = \frac{\Upsilon}{\Sigma}$ which is independent of α and prop. to $R^{3/4}$

When a gas is fully ionized $\kappa \simeq 0.4 < \kappa_R$

So κ_R is valid for

$$\left(\frac{R}{R_{sch}}\right) \geq (2.6\times10^4)\left(\frac{\eta}{0.1}\right)^{-2/3}\left(\frac{\dot M}{\dot M_{Edd}}\right)^{2/3} y^{8/3}$$