Timescales & Stability

Viscous timescale: $t_{visc} \sim \frac{R^2}{\nu} \sim \frac{R}{V_R}$ timescale on which matter diffuses through the disk b/c of viscous torques

Dynamical timescale $t_\phi \sim \frac{R}{V_\phi} \sim \Omega_K^{-1}$

Hydrostatic timescale $t_z = \frac{H}{c_s} = \frac{c_s}{V_\phi}\frac{R}{c_s} \sim t_\phi$ b/c of thin disks

Thermal timescale: $t_{th} = \frac{\text{(heat content per unit disk area)}}{\text{(dissipation rate per unit disk area)}}$

Heat content per unit volume $\sim P_{gas} \sim \frac{\rho k T}{\mu m_p} \sim \rho c_s^2$

$$\text{so, } t_{th} = \frac{\Sigma c_s^2}{D(R)}$$

$$D(R) = \frac{1}{2}\nu\Sigma(R\Omega')^2 = \frac{9}{8}\nu\Sigma\frac{GM}{R^3} \text{ for a Keplerian Disk}$$

$$\therefore t_{th} \sim \frac{\Sigma c_s^2}{\nu\Sigma\frac{GM}{R^3}} \sim \frac{R^3 c_s^2}{\nu GM} \sim \frac{c_s^2}{\nu\Omega_K^2} \sim \frac{R^2 c_s^2}{\nu R^2\Omega_K^2} \sim \frac{R^2}{\nu}\frac{c_s^2}{V_\phi^2} \sim \mathcal{M}^{-2} t_{visc}$$

$$\text{But } t_{visc} \sim \frac{R^2}{\nu} \sim \frac{1}{\alpha}\frac{R\,R}{c_s H} = \frac{1}{\alpha}\frac{R}{H}\frac{R}{V_\phi}\frac{V_\phi}{c_s} = \alpha^{-1}\mathcal{M}^2 t_\phi$$

$$\text{so, } t_{th} \sim \alpha^{-1} t_\phi \text{ or } t_\phi \sim \alpha t_{th}$$

$$\therefore \quad t_\phi \sim t_z \sim \alpha t_{th} \sim \alpha\left(\frac{H}{R}\right)^2 t_{visc}$$

If $\alpha \leq 1$ then we have the following hierarchy of timescales

$$t_\phi \sim t_z \leq t_{th} << t_{visc}$$

For the outer disk solution:

$$t_\phi \sim t_z \sim \alpha t_{th} \sim (77.5s)\left(\frac{M}{M_\odot}\right)^{-1/2}\left(\frac{R}{10^{10}cm}\right)^{3/2}$$

$$t_{visc} \sim (5.3\times10^5 s)\,\alpha^{-4/5}\left(\frac{\dot{M}}{10^{16}\frac{g}{s}}\right)^{-3/10}\left(\frac{M}{M_\odot}\right)^{1/4}\left(\frac{R}{10^{10}cm}\right)f^{-6/5}$$

$\therefore$ Mass transfer instabilities propogate through the disk on a timescale of days to weeks, wheras dynamical & thermal instabilities manifest on the order of minutes.

Thermal instability will occur if the cooling rate cannot keep up w/ the heating rate & the disk will puff up. If $\alpha \sim$ const. this happens only when the disk is optically thin.

A viscous instability occurs if $\frac{\partial \dot{M}}{\partial \Sigma} < 0$ and the disk breaks up into rings. Both of these instabilities occur in rad-pressure dominated α-disks. BUT...?

At each radius R, the intensity of emerging radiation is

$$I_\nu(R) = B_\nu(T(R))$$

where B_ν is the Planck/BB function

$$B_\nu = \frac{2h\nu^3/c^2}{e^{h\nu/(kT(R))}-1}$$

An observer at a distance D w/ a l.o.s. at an angle i will measure a flux

$$F_\nu = \int_{R_*}^{R_{out}} I_\nu \, d\Omega(R) \quad \text{where } d\Omega(R) = \frac{2\pi R\, dR \cos i}{D^2}$$

$$\therefore F_\nu = \frac{4\pi h\nu^3 \cos i}{c^2 D^2} \int_{R_*}^{R_{out}} \frac{R\,dR}{e^{h\nu/kT(R)} - 1}$$

In the low-freq. (Rayleigh-Jeans limit), defined by $h\nu \ll kT(R_{out})$

$$B_\nu^{RJ} \simeq \frac{2kT(R)\,\nu^2}{c^2}$$

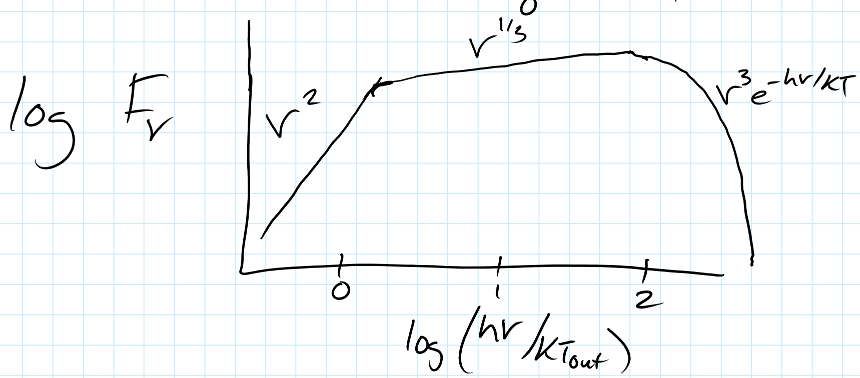
$$\therefore F_\nu^{RJ} \propto \nu^2$$

In the opposite limit, $h\nu \gg kT(R_*)$ the Planck function has the Wien shape, $B_\nu^W \simeq \frac{2h\nu^3}{c^2} e^{-h\nu/kT}$ so $F_\nu^W \propto \nu^3 e^{h\nu/kT}$

In b/w these limits $T \propto R^{-3/4}$, so define a variable

$$\eta \equiv \frac{h\nu}{kT(R)} \simeq \frac{h\nu}{kT(R_*)} \left(\frac{R}{R_*}\right)^{-3/4}$$

$$\text{so } F_\nu \propto \nu^{1/3} \int_0^\infty \frac{\eta^{5/3}\, d\eta}{e^\eta - 1} \propto \nu^{1/3}$$



But in BH/NSs:

irradiation
relativity
Comptonization
turbulence
instabilities