ADAFs - or Advection Dominated Accretion Flows

The thin disk solution relies on the fact that the disk can radiate away all the viscous energy. But, at high $\dot{M}/\dot{M}_{Edd}$ where radiation pressure dominates, $H/R \sim 1$ and this assumption can break down, and the viscous energy may be advected w/ the flow into the BH, reducing the radiative efficiency.

Also, at low $\dot{M}/\dot{M}_{Edd}$ the disk density may be so low that radiative cooling is inefficient & $H/R \sim 1$ & advection is important.

In both cases, the vert. integrated hydro eqn's are (for steady state)

$$\frac{d}{dR}(\rho R H v_r) = 0$$

note that this may be $\neq \Omega_K$

$$v_r \frac{dv_r}{dR} - \Omega^2 R = -\Omega_K^2 R - \frac{1}{\rho}\frac{d}{dR}(\rho c_s^2)$$

$$\frac{v_r d(\Omega R)}{dR} = \frac{1}{\rho R H}\frac{d}{dR}\left(\nu \rho H R^3 \frac{d\Omega}{dR}\right)$$

Energy eqn:

$$q^+ - q^- = \rho\nu(R\Omega')^2 - q^-$$

q^- = radiative cooling per unit vol.

$$q_{adv} = q^+ - q^- = \rho v_R T \frac{ds}{dR}$$

s = specific entropy of the gas

$$q_{adv} = f\rho\nu(R\Omega')^2$$

f = ratio of advected energy over heat generated

So, $q^+ \simeq q^- \gg q^{adv}$ cooling can keep up & thin disks are valid

$q^{adv} \simeq q^+ \gg q^-$ ADAF; viscous energy is stored in the gas, radiatively inefficient

$-q^{adv} \simeq q^- \gg q^+$ Bondi accretion

Assuming Newtonian gravity, $f \sim 1$, $f \neq f(R)$ & $\alpha^2 \ll 1$, can find a self-similar solution (far from boundaries)

define γ = ratio of specific heats

$$\frac{V_r}{V_{ff}} \simeq -\left(\frac{\gamma-1}{\gamma-\frac{5}{9}}\right)\alpha, \quad \frac{\Omega}{\Omega_K} \simeq \left[\frac{2(5/3-\gamma)}{3(\gamma-\frac{5}{9})}\right]^{1/2}, \quad \frac{C_S^2}{V_{ff}^2} \simeq \frac{2}{3}\left(\frac{\gamma-1}{\gamma-\frac{5}{9}}\right)$$

where $V_{ff} = \sqrt{\frac{GM}{R}}$

For 1) $\alpha \sim 0.2-0.3$, the $V_r \gtrsim 0.1\, V_{ff}$. Rapid accretion

2) Rotation is sub-Keplerian & $\to 0$ as $\gamma \to \frac{5}{3}$

3) Temperatures are large, so $H \sim \frac{c_s}{\Omega_K} \sim \frac{V_{ff}}{\Omega_K} \sim R$

4) Gas has a positive Bernoulli parameter, its unbound and could be lost to outflows

5) $\frac{ds}{dR} < 0$ and this means gas is convectively unstable

running simulations

In the high $\dot{M}$, optically thick case, this theory has been used to 'correct' the thin disk equations into a 'slim' disk model. Few observational tests. Jury still out.

Focus on low-$\dot{M}$ ADAFs which are relevent to LLAGNs, Sgr A* and other weakly accreting sources. Really need simulations.

These are two-T flows and optically thin.

Suppose viscous dissipation heats particles equally by mass, ie mostly to ions ; $\sim \frac{1}{2000}$ to the e^-.

But energy transferred to e^- are 2-body processes & slow under these conditions. The interaction time $\propto \frac{1}{v_p} \propto T_p^{-1/2}$

$\therefore$ Ions approach the virial temp. $\sim 10^{12}$ K

e^- temp $\sim kT \sim 50-200$ keV

For constant α, need accretion rates $\frac{\dot M}{\dot M_{Edd}} \lesssim \alpha^2$ for this solution.

Assuming $f \to 1$; $P_{mag} = 0.5\, P_{gas}$; $\eta = 0.1$

$$V_r \simeq \left(-1.1\times 10^{10}\ \tfrac{cm}{s}\right)\alpha \left(\frac{r}{R_{sch}}\right)^{-1/2}$$

$$\Omega \simeq 2.9\times 10^{4}\left(\frac{M}{M_\odot}\right)^{-1}\left(\frac{R}{R_{sch}}\right)^{-3/2} s^{-1}$$

$$c_s^{2} \simeq \left(1.4\times 10^{20}\right)\left(\frac{R}{R_{sch}}\right)^{-1} cm^2\, s^{-2}$$

$$n_e \simeq \left(6.3\times 10^{19}\,cm^{-3}\right)\left(\frac{M}{M_\odot}\right)^{-1}\left(\frac{\dot M}{\dot M_{Edd}}\right)\alpha^{-1}\left(\frac{R}{R_{sch}}\right)^{-3/2}$$

$$B \simeq \left(7.8\times 10^{8}\,G\right)\alpha^{-1/2}\left(\frac{M}{M_\odot}\right)^{-1/2}\left(\frac{\dot M}{\dot M_{Edd}}\right)^{1/2}\left(\frac{R}{R_{sch}}\right)^{-5/4}$$

$$P = \left(1.7\times 10^{16}\,g\,cm^{-1}s^{-2}\right)\alpha^{-1}\left(\frac{M}{M_\odot}\right)^{-1}\left(\frac{\dot M}{\dot M_{Edd}}\right)\left(\frac{R}{R_{sch}}\right)^{-5/4}$$

$$q^{+} \simeq \left(5\times 10^{21}\,erg/s\,cm^{3}\right)\left(\frac{M}{M_\odot}\right)^{-2}\left(\frac{\dot M}{\dot M_{Edd}}\right)\left(\frac{R}{R_{sch}}\right)^{-4}$$

$$\tau \simeq 24\,\alpha^{-1}\left(\frac{\dot M}{\quad}\right)\left(\frac{R}{\quad}\right)^{-1/2}$$

$$\tau_{es} \simeq 24\,\alpha^{-1} \left(\frac{\dot{M}}{\dot{M}_{Edd}}\right)\left(\frac{R}{R_{sch}}\right)^{-1/2}$$