Theory Notes for AGNs

1) The inner radius of accretion disks is limited to the innermost stable orbits which are derived from G.R. & depend on spin (a)

Using $G=c=1$, the natrual unit of length used in AGN studies

is $r_g = \frac{GM}{c^2} = M \qquad (0M \leq a \leq 0.998M)$

The innermost boud circular orbits are

$$r_{ms} = M\left\{3 + Z_2 \mp \left[(3-Z_1)(3+Z_1+2Z_2)\right]^{1/2}\right\}$$

where upper sign is prograde orbits
lower sign is retrograde orbits

$$Z_1 = 1 + \left(1 - \frac{a^2}{M^2}\right)^{1/3}\left[\left(1+\frac{a}{M}\right)^{1/3} + \left(1-\frac{a}{M}\right)^{1/3}\right]$$

$$Z_2 = 3\left(\frac{a^2}{M^2} + Z_1^2\right)^{1/2}$$

For $a=0$ (Schwarzschild) $r_{ms} = 6M = 6\frac{GM}{c^2} = 3R_{Sch.}$

$a = M$ (max. Kerr BH), $r_{ms} = \begin{cases} M & \text{prograde} \\ 9M & \text{retrograde} \end{cases}$

The binding energy of the marginally stable circular orbit increases as $a \to M$. So, accretion efficiency $\eta = 0.057 \to 0.42$

2) Effects of GR on α-disk theory

Recall, we had the dissipation per unit disk face area

$$D(R) = \frac{3GM\dot{M}}{8\pi R^3}\underbrace{\left(1 - \left(\frac{R_*}{R}\right)^{1/2}\right)}$$

↳ changed this term to absorb the GR corrections now, $D(R) = \frac{3GM\dot{M}}{8\pi R^3} R_R(x)$ where $x = \frac{R}{r_g}$

also, $\nu\Sigma = \frac{\dot{M}}{3\pi}\left(1 - \left(\frac{R_*}{R}\right)^{1/2}\right)$ from cons. of ang. mom.

becomes $\nu\Sigma = \frac{\dot{M}}{3\pi} R_T(x)$

Hydrostatic equilibrium in the vertical direction is

$$\frac{dP}{dz} = \rho g_z = -\frac{GM\rho z}{R^3} R_z(x)$$

where: $R_R(x) = \frac{C(x)}{B(x)}$, $R_T(x) = \frac{C(x)}{A(x)}$; $R_z(x) = x^{-1}\left[L^2 - a_*^2(E_\infty - 1)\right]$

$A(x) = 1 - \frac{2}{x} + \frac{a_*^2}{x^2}$, $B(x) = 1 - \frac{3}{x} + \frac{2a_*}{x^{3/2}}$ and

$$C(x) = 1 - \frac{y_{ms}}{y} - \frac{3a_*}{2y}\ln\left(\frac{y}{y_{ms}}\right) - \frac{3(y_1 - a_*)^2}{y y_1 (y_1 - y_2)(y_1 - y_3)}\ln\left(\frac{y - y_1}{y_{ms} - y_1}\right)$$
$$- \frac{3(y_2 - a_*)^2}{y y_2 (y_2 - y_1)(y_2 - y_3)}\ln\left(\frac{y - y_2}{y_{ms} - y_2}\right) - \frac{3(y_3 - a_*)^2}{y y_3 (y_3 - y_1)(y_3 - y_2)}\ln\left(\frac{y - y_3}{y_{ms} - y_3}\right)$$

where $y = \sqrt{x}$; y_{ms} = value of y at r_{ms}

$y_{1,2,3}$ = 3 roots of $y^3 - 3y + 2a_* = 0$

$$L(x) = \frac{x^{1/2}(1 - 2a_* x^{-3/2} + a_*^2 x^{-2})}{B^{1/2}(x)}, \quad E_\infty(x) = \frac{1 - 2/x + a_* x^{-3/2}}{B^{1/2}(x)}$$