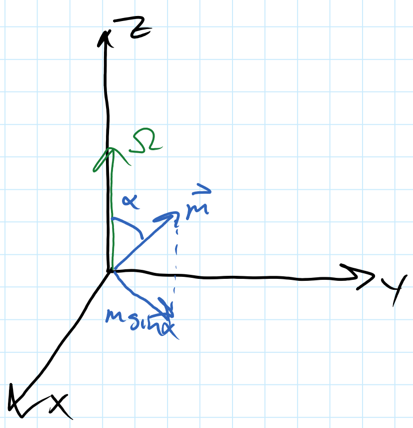
Magnetic Dipole Radiation from Pulsars [SI]

The most famous pulsar lies in the Crab nebula, the site of a SN explosion in 1054 (observed by Chinese astronomers). The nebula consists of outwardly expanding filaments emitting optical line radiation (shocked gas) and an amorphous region emitting synchroton radiation. The period of the Crab pulsar is 0.0331 s & is increasing w/ $\dot{P} \sim 10^{-15}$ s s^{-1}. The pulsar is spinning down. What is happening to the rotation energy?

Assume a NS rotates uniformly @ a freq. Ω & possesses a mag. dipole moment $\vec{m}$ oriented at an angle α to the rotation axis. The rotation is assumed to be sufficiently slow that nonspherical distortions due to rotation can be ignored to lowest order.



Let $\vec{m}$ lie in the xz plane at $t=0$

$$\vec{m} = |\vec{m}|\cos\alpha\,\hat{z} + |\vec{m}|\sin\alpha\left[\cos(\Omega t)\hat{x} + \sin(\Omega t)\hat{y}\right]$$

The magnitude of the dipole moment is

$$m = \frac{4\pi R^3 B}{\mu_0}$$ where R is the radius of the NS.

A rotating magnetic dipole can be thought of as the superposition of 2 $\perp$ oscillating dipoles w/ one out of phase by 90°. In general,

$$\vec{S} = \frac{1}{\mu_0}(\vec{E}\times\vec{B}) = \frac{1}{\mu_0}\left[(\vec{E}_1+\vec{E}_2)\times(\vec{B}_1+\vec{B}_2)\right]$$

$$= \frac{1}{\mu_0}\left[(\vec{E}_1\times\vec{B}_1)+(\vec{E}_2\times\vec{B}_2)+(\vec{E}_1\times\vec{B}_2)+(\vec{E}_2\times\vec{B}_1)\right]$$

$$= \vec{S}_1 + \vec{S}_2 + \text{cross-terms}$$

In this case, the field of 1 & 2 are 90° out of phase, so the cross-terms → 0 in the time averaging. The total power radiated is just the sum of the 2 individual powers.

∴ A rotating magnetic dipole will radiate a time-averaged power of $P = \frac{\mu_0 m_0^2 \omega^4}{6\pi c^3}$ where m_0 = max. value of dipole moment

For the case of the pulsar, $\omega = \Omega$ & since only the rotating comp. of $\vec{m}$ will radiate, $m_0 = m\sin\alpha$

$$\therefore P = \frac{\mu_0 m^2 \sin^2\alpha\, \Omega^4}{6\pi c^3} = \cancel{\mu_0}\frac{16\pi^{\cancel{2}}R^6B^2}{\mu_0^{\cancel{2}}}\frac{\sin^2\alpha}{\cancel{6\pi}c^3}\Omega^4$$

$$P = \frac{8\pi R^6 B^2 \Omega^4 \sin^2\alpha}{3\mu_0 c^3}$$