The energy carried away by the radiation originates from the rotational energy of the NS: $E = \frac{1}{2} I \Omega^2$ where I is the moment of inertia

$$\therefore \dot{E} = I \Omega \dot{\Omega}$$

Since $\dot{E} < 0$, $\dot{\Omega} < 0$ and the pulsar slows down.

Define a characteristic age T at the present time by

$$T \equiv -\left(\frac{\Omega}{\dot{\Omega}}\right)_0 = -\frac{2E}{\dot{E}} = \frac{-I\Omega_0^2}{\left(-\frac{8\pi R^6 B^2 \Omega_0^4 \sin^2\alpha}{3\mu_0 c^3}\right)}$$

$$= \frac{3\mu_0 c^3 I}{8\pi R^6 B^2 \Omega_0^2 \sin^2\alpha}$$

Now, lets look at the time evol. of Ω:

$$\Omega \frac{d\Omega}{dt} = \frac{\dot{E}}{I} = -\frac{8\pi R^6 B^2 \Omega^4 \sin^2\alpha}{3\mu_0 c^3 I} = \frac{-\Omega_0^2}{\Omega_0^2} \frac{8\pi R^6 B^2 \sin^2\alpha}{3\mu_0 c^3 I}\Omega^4$$

$$= \frac{-\Omega^4}{\Omega_0^2 T}$$

$$\int_{\Omega_i}^{\Omega} \Omega^{-3} d\Omega = -\frac{1}{\Omega_0^2 T}\int_0^t dt$$

$$\vdots$$

$$\Omega = \Omega_i \left(1 + \frac{2\Omega_i^2}{\Omega_0^2}\frac{t}{T}\right)^{-1/2}$$

Set $\Omega = \Omega_0$, then this gives the present age of the pulsar

$$t = \frac{T}{2}\left(1 - \frac{\Omega_0^2}{\Omega_i^2}\right) \simeq \frac{T}{2} \quad \text{for } \Omega_0 \ll \Omega_i$$

This estimate of age does not depend on the detailed properties of the underlying NS, only on the behavior of $\Omega(t)$ due to magnetic dipole radiation.

For the Crab pulsar T was measured to be 2486 yr (in 1972), implies an age from mag. dipole rad'n, of 1243 yr (actually 918)

For a $1.4 M_\odot$, $R = 12$ km NS, $I = 1.4 \times 10^{45}\ \mathrm{g\,cm^2}$

$$\dot{E} = I\Omega\dot{\Omega} = 6.4 \times 10^{38}\ \mathrm{erg\,s^{-1}}$$

Observed rad'n $\bar{c}$ KE losses $\simeq 5 \times 10^{38}$ erg s^{-1}

(radio pulse $\sim 10^{31}$ erg s^{-1})

Using this $\dot{E}$ from the Crab $\bar{c}$ adopting the mag. dipole model

$$B \simeq 5 \times 10^{12}\ G \quad (\sin\alpha = 1)$$

Such a value arises naturally from the collapse of a main sequence star w/ a typical 'frozen-in' surface field of 100 G. The decrease in radius by a factor of 10^5 leads to an increase in B by a factor of $\sim 10^{10}$.

For magnetic dipole model $\bar{c}$ uniform sphere $(I = 2MR^2/5)$

leads to an increase in B by a factor of ~10.

For magnetic dipole model & uniform sphere ($I = 2MR^2/5$)

$$B = -\left(\frac{3\mu_0 c^3 M\dot{\Omega}}{20\pi\Omega^3 R^4}\right)^{1/2}$$

Using $\dot{\Omega} = -\frac{\dot{P}\Omega^2}{2\pi}$, $B = \left(\frac{3\mu_0 c^3 M}{80\pi^3 R^4}\right)(P\dot{P})^{1/2}$

$$\simeq 3\times10^{11}(P\dot{P})^{1/2}\ G$$

Braking Index

For any power-law deceleration model, we write

$$\dot{\Omega} = -(\text{constant})\,\Omega^n = -K\Omega^n$$

n = braking index (=3 ~~for~~ the mag-dipole model)

differentiating $\ddot{\Omega} = -nK\Omega^{n-1}\dot{\Omega}$ & then divide by the $\dot{\Omega}$ expression

$$\frac{\ddot{\Omega}}{\dot{\Omega}} = \frac{\cancel{-}nK\Omega^{n-1}\dot{\Omega}}{\cancel{-}K\Omega^n}$$

$$\frac{\ddot{\Omega}}{\dot{\Omega}^2} = \frac{n}{\Omega} \rightarrow n = \frac{\Omega\ddot{\Omega}}{\dot{\Omega}^2} = 2 - \frac{P\ddot{P}}{\dot{P}^2}$$

Thus, n can be measured

$n = 2.515$ Crab

$n = 2.837$ PSR B1509-58

$n = 1.81$ PSR B0540

$n = 3$ PSR J1119

Magnetic breaking is not the whole story, but B, α or I could all vary w/ time.

Accretion onto Magnetized NSs [SI]

The NS B-field is roughly a dipole so $|B| \propto \frac{1}{r^3}$

ie, $B \simeq \left(\frac{R_*}{r}\right)^3 B_s$ where B_s is field strength of surface of NS w/ radius R_*

This B-field will have an associated mag. pressure

$$P_{mag} \simeq \frac{B^2}{2\mu_0} \simeq \left(\frac{B_s^2}{2\mu_0}\right)\left(\frac{R_*}{r}\right)^6$$

Consider spherical accretion from ∞, so the gas is falling radially at its free-fall speed $v = \left(\frac{2GM_*}{r}\right)^{1/2}$

It will exert a ram pressure $P_{ram} = \rho v^2$ radially inwards

[P_{ram} = mom. density × velocity = momentum flux]

When $P_{ram} = P_{mag}$, the magnetic field will become dynamically important to the accreting gas.

$$\rho v^2 = \frac{B_s^2}{2\mu_0}\left(\frac{R_*}{r_M}\right)^6$$

$$\rho v^2 = \frac{B_s^2}{2\mu_0}\left(\frac{R_*}{r_M}\right)^6$$

where r_M is the Alfvén radius. From mass continuity,

$$\dot{M} = 4\pi r^2 \rho v:$$

$$\frac{\dot{M}}{4\pi r_M^2} \times \frac{(2GM_*)^{1/2}}{r_M^{1/2}} = \frac{B_s^2}{2\mu_0}\frac{R_*^6}{r_M^6}$$

$$r_M = \left(\frac{2\pi^2}{G\mu_0^2}\right)^{1/7}\left(\frac{B_s^4 R_*^{12}}{M_* \dot{M}^2}\right)^{1/7}$$

For an Eddington limited accreting NS w/ $\eta = 0.1$, $B_s \simeq 10^{12}$ G

; $R_* = 10$ Km, $r_M \simeq 10^3$ Km $= 100\, R_s$

$(r_M \lesssim R_*$ for $B_* \lesssim 10^9$ G$)$