Within r_M, matter will accrete along the field lines and only a fraction ($f \sim \frac{R_*}{r_M} \sim 10^{-1}$) of the star accumulates matter

→ X-ray pulsars w/ periods $1\,s < P_{spin} < 10^3\,s$

(Considerations of a thin disk interacting w/ the magnetosphere give results within a factor of 2 of the r_M given above)

The accretion disk can exert a torque on the NS and spin it up if $\Omega_* < \Omega(r_M)$

Assume Keplerian rotation @ r_M ; that $\Omega^* \ll \Omega(r_M)$

the rate of inward transfer of ang. mom. @ radius r_M is

$$\dot{M} v_\phi r_m = \dot{M} r_M^2 \Omega_K(r_M)$$

This is absorbed by the star

$$I\dot{\Omega}_* = \dot{M} r_m^2 \Omega_K(r_M) = \dot{M}(GM_* r_M)^{1/2}$$

but $L = \frac{GM\dot{M}}{R_*}$ (energy still released at R_*)

$$I 2\pi \dot{f}_* = \frac{R_* L}{GM_*}(GM_*)^{1/2}\left(\frac{2\pi^2}{G\mu_0^2}\right)^{1/14}\left(\frac{B_*^4 R_*^{12}}{M_* \dot{M}^2}\right)^{1/14}$$

⋮

$$\dot{f}_* = \frac{1}{2\pi I}\left(\frac{2\pi^2}{G^6 \mu_0^2}\right)^{1/4}\left(B_*^2 M_*^{-3} R_*^{12}\right)^{1/7} L^{6/7}$$

For NS parameters $\dot{f} \sim 10^{-12}\ Hz\ s^{-1}$

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Spin up ceases when $\Omega_* \simeq \Omega_K(r_M)$

$$\text{or} \quad \frac{2\pi}{P_*} = \left(\frac{GM_*}{r_M^3}\right)^{1/2} \rightarrow P_* = 2\pi\left(\frac{r_M^3}{GM_*}\right)$$

$$\rightarrow P_* = \frac{2\pi}{(GM_*)^{1/2}}\left(\frac{2\pi^2}{G\mu_0^2}\right)^{3/14}\left(\frac{R_*^{12}}{M_*\dot{M}^2}\right)^{3/14} B_*^{6/7}$$

as $\dot{M}\uparrow$ $P_*\downarrow$ to higher rotation rate

but $B_*\uparrow$ $P_*\uparrow$ resists spinup

For an Eddington limited NS,

$$P_{min} \simeq 2\left(\frac{B_*}{10^9 G}\right)^{6/7} \text{ms}$$

$\rightarrow$ ms pulsars are spun-up older systems

Combine this eq'n w/ $B_* = 3\times10^{11}(P\dot{P})^{1/2}$ G to get 'spin-up' line in $P\dot{P}$ diagram