In general, we need to relate Θ' to angles in lab frame which is very difficult so examine 2 special cases

1) Acc'n $\parallel$ to velocity: $\Theta' = \theta'$

$$\mu'^2 = \frac{(\mu-\beta)^2}{(1-\beta\mu)^2} = \frac{\mu^2 - 2\beta\mu + \beta^2}{(1-\beta\mu)^2} = \frac{1-\sin^2\theta - 2\beta\mu + \beta^2}{(1-\beta\mu)^2}$$

$$1 - \sin^2\theta' = \frac{1-\sin^2\theta - 2\beta\mu + \beta^2}{(1-\beta\mu)^2}$$

$\vdots$ ASA

$$\sin^2\theta' = \frac{\sin^2\theta}{\gamma^2(1-\beta\mu)^2}$$

$$\therefore \frac{dP_{\parallel}}{d\Omega} = \frac{q^2}{4\pi c^3} \frac{a_{\parallel}^2 \sin^2\theta}{(1-\beta\mu)^4}$$

2) Acc'n $\perp$ to velocity:

$\cos\Theta' = \sin\theta'\cos\phi'$, so that $\cos^2\Theta' = \sin^2\theta'\cos^2\phi'$

but $\cos\phi' = \cos\phi$ $\therefore$ from above $\sin^2\Theta' = 1 - \dfrac{\sin^2\theta\cos^2\phi}{\gamma^2(1-\beta\mu)^2}$

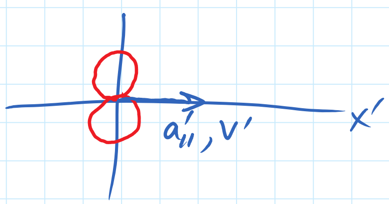
$$\frac{dP_{\perp}}{d\Omega} = \frac{q^2 a_{\perp}^2}{4\pi c^3} \frac{1}{(1-\beta\mu)^4}\left[1 - \frac{\sin^2\theta\cos^2\phi}{\gamma^2(1-\beta\mu)^2}\right]$$

In both cases, when $\gamma \gg 1$, $(1-\beta\mu)$ becomes small when $\theta \sim 0$. Can approx. $\mu \simeq 1 - \frac{\theta^2}{2}$; $\beta = \left(1 - \frac{1}{\gamma^2}\right)^{1/2} \simeq 1 - \frac{1}{2\gamma^2}$

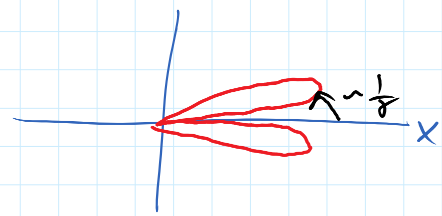
then $$(1-\beta\mu) \simeq \frac{1+\gamma^2\theta^2}{2\gamma^2}$$

and $$\frac{dP_{\parallel}}{d\Omega} \simeq \frac{16 q^2 a_{\parallel}^2}{\pi c^3} \gamma^{10} \frac{\gamma^2\theta^2}{(1+\gamma^2\theta^2)^6}$$

'Rest'-frame

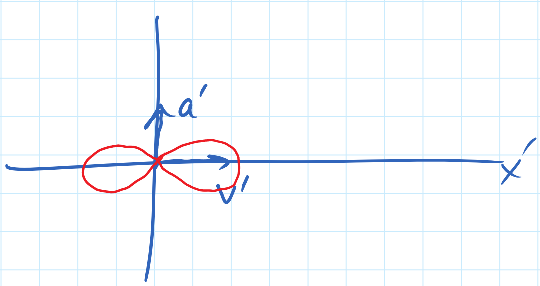


Lab frame

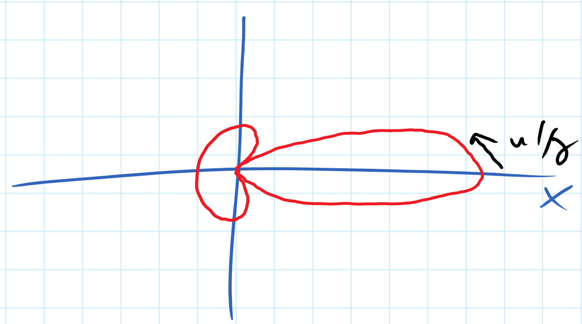


$$\frac{dP_{\perp}}{d\Omega} \simeq \frac{4 q^2 a_{\perp}^2}{\pi c^3} \gamma^8 \frac{\left(1 - 2\gamma^2\theta^2 \cos 2\phi + \gamma^4\theta^4\right)}{(1+\gamma^2\theta^2)^6}$$

'Rest' Frame



Lab Frame



Both depend on θ solely through $\gamma\theta \rightarrow$ the peak is for angles $\theta \sim \frac{1}{\gamma}$

Bremsstrahlung or Free-Free Emission (Classical analysis)

Radiation due to an electron accelerated in the Coulomb field of an ion. Note that there is no dipole radiation from the collision of like particles (e.g., $e^- - e^-$)

Proof: Consider a system w/ N particles w/ charge-to-mass ratio K, i.e. $K = \frac{q_1}{m_1} = \frac{q_2}{m_2} = \cdots = \frac{q_N}{m_N}$

Then $\vec{d} = \sum q_i \vec{r}_i = q_1 \vec{r}_1 + q_2 \vec{r}_2 + \cdots + q_N \vec{r}_N$

$$= K m_1 \vec{r}_1 + K m_2 \vec{r}_2 + \cdots + K m_N \vec{r}_N$$

$$= K \sum m_i \vec{r}_i$$

But $\frac{\sum m_i \vec{r}_i}{\sum m_i} = \vec{r}_{c.o.m}$ or $\sum m_i \vec{r}_i = \left(\sum m_i\right) \vec{r}_{c.o.m}$

$$\therefore \ddot{\vec{d}} = K\left(\sum m_i \ddot{\vec{r}}_i\right) = K\left(\sum m_i\right) \ddot{\vec{r}}_{c.o.m.}$$

For an isolated system, $\dot{\vec{r}}_{c.o.m} = \text{const}$ so

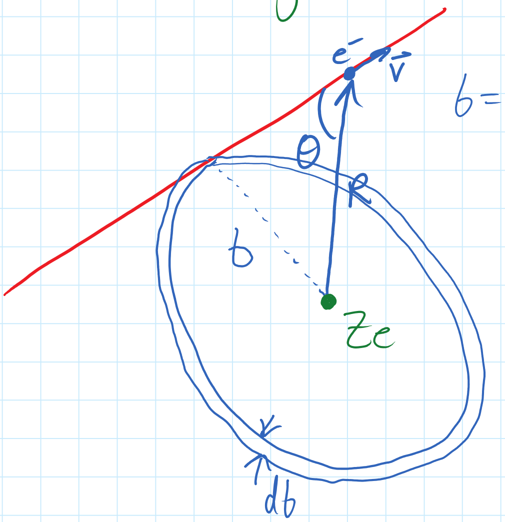
$$\ddot{\vec{d}} = 0 \quad \blacksquare$$

Consider 2 diff. particles: an e^- & an ion w/ charge

$+Ze$; e^- is the primary radiater

1) Emission from 1 single-speed e^-

Assume e^- moves quickly enough so that deviation from a straight-line is small



b = impact parameter

Charge of electron $-e$

$$\therefore \vec{d} = -e\vec{R}$$

$$; \ \ddot{\vec{d}} = -e\dot{\vec{v}}$$