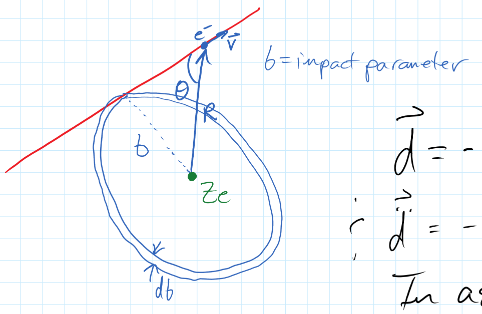

$\vec{d} = -eR$

$\therefore \ddot{\vec{d}} = -e\dot{\vec{v}}$

The acc'n will vary as the e^- whizzes by so the freq. of the dipole radiation will be broad.

In astrophys, we are mostly interested in the spectrum of the emission $\therefore$ Analyze this in freq. space.

Take F.T. of both sides

$$\underbrace{-\omega^2 \hat{d}(\omega)}_{\text{from before}} = -\frac{e}{2\pi} \int_{-\infty}^{+\infty} \dot{\vec{v}} e^{i\omega t} dt$$

The characteristic timescale of the interaction is $\tau = \frac{b}{v}$.

So, what is the contribution to the integral for the high freq. case $1 \ll \omega\tau$?

- v. small b/c integral oscillates v. quickly over timescales of interest

What about for $1 \gg \omega\tau$?

$$e^{i\omega t} = \cos\omega t + i\sin\omega t \simeq 1 \quad \text{when} \quad \omega t \ll 1$$

$$\hat{d}(\omega) \simeq \begin{cases} \frac{e}{2\pi\omega^2} \Delta\vec{v} & \omega\tau \ll 1 \\ 0 & \omega\tau \gg 1 \end{cases}$$

where $\Delta\vec{v}$ is the change of velocity during the collision

The energy spectrum of the radiation emitted by the e^- is $\frac{dW}{d\omega} = \begin{cases} \frac{2e^2}{3\pi c^3} |\Delta\vec{v}|^2 & \omega\tau \ll 1 \end{cases}$

e^- is $$\frac{dW}{d\omega} = \begin{cases} \frac{2e^2}{3\pi c^3}|\Delta v|^2 & \omega\tau \ll 1 \\ 0 & \omega\tau \gg 1 \end{cases}$$

Now calculate $\Delta\vec{v}$. In the small-angle scattering regime, Δv is mostly normal (any $\Delta v_\parallel$ integrates to zero over the path)

Integrate $\Delta\vec{v}_\perp$ over the path

$$\vec{F}_\perp = m\left(\frac{d\vec{v}_\perp}{dt}\right) = \frac{Ze^2}{R^2}\cos\theta = \frac{Ze^2\cos\theta}{\left(\sqrt{b^2+v^2t^2}\right)^2} = \frac{Ze^2 b}{(b^2+v^2t^2)^{3/2}}$$

$$\Delta v_\perp = \frac{Ze^2}{m}\int_{-\infty}^{+\infty}\frac{b\,dt}{(b^2+v^2t^2)^{3/2}} = \frac{Ze^2}{m}b\int_{-\infty}^{+\infty}\frac{dt}{v^3\left(\frac{b^2}{v^2}+t^2\right)^{3/2}}$$

$$= \frac{Ze^2 b}{mv^2}\int_{-\infty}^{+\infty}\frac{dt}{\left(\frac{b^2}{v^2}+t^2\right)^{3/2}}$$

let $t = \frac{b}{v}\tan\theta$

$dt = \frac{b}{v}\sec^2\theta\, d\theta$

when $t = +\infty$, $\theta = \frac{\pi}{2}$

$t = -\infty$, $\theta = -\frac{\pi}{2}$

...ASA...

$$\Delta v_\perp = \frac{2Ze^2}{mbv}$$

$\therefore$ The single-particle bremsstrahlung spectrum for an impact parameter b is:

$$\frac{dW(b)}{d\omega} = \begin{cases} \frac{8Z^2e^6}{3\pi c^3 m^2 v^2 b^2} & b \ll \frac{v}{\omega} \\ 0 & b \gg \frac{v}{\omega} \end{cases}$$

Now, the total spectrum for a mono-energetic dist'n of e^- scattering off a population of ions:

ion density: n_i (cm^{-3})

e^- density: n_e (cm^{-3})

flux of e^- incident on one ion: $n_e v$ $(cm^{-2}s^{-1})$

$\therefore$ the rate of e^- passing through an annulus around a single ion of width db at b is $n_e v 2\pi b\,db$

$\Rightarrow$ The total emission per unit time per unit volume, per freq. interval is

$$\frac{dW}{d\omega\,dt\,dV} = n_e n_i 2\pi v \int_{b_{min}}^{\infty} \frac{dW(b)}{d\omega}\, b\, db$$

To a good approximation we can use the low freq. version of $\frac{dW(b)}{d\omega}$ and so

$$\frac{dW}{d\omega\,dV\,dt} = \frac{16e^2}{3c^3m^2v} n_e n_i Z^2 \int_{b_{min}}^{b_{max}} \frac{db}{b} = \frac{16e^2}{3c^3m^2v} n_e n_i Z^2 \ln\left(\frac{b_{max}}{b_{min}}\right)$$

The value of b_{max} is uncertain, but is order of $\frac{v}{\omega}$; since only comes in in the ln term, set $b_{max} = \frac{v}{\omega}$

For b_{min}, use the Uncertainty Principle: $\Delta x \Delta p \gtrsim \hbar$

let $\Delta x \sim b$; $\Delta p \sim mv$

$$\therefore b_{min} \simeq \frac{\hbar}{mv}$$

Get results to within an order of mag. w/ these limits