For any regime the exact results are conviently stated in terms of a correction factor $g_{ff}(v,\omega)$
- its called the Gaunt factor

$$\frac{dW}{d\omega\, dt\, dV} = \frac{16\pi e^6}{3\sqrt{3}c^3 m^2 v} n_e n_i Z^2 g_{ff}(v,\omega)$$

where $g_{ff}(v,\omega)$ for the classical result is

$$g_{ff}(v,\omega) = \frac{\sqrt{3}}{\pi} \ln\left(\frac{b_{max}}{b_{min}}\right)$$

This result is for particles w/ the same energy. Now want to calculate the spectrum produced by e^- w/ a dist'n of energies. Here, we calculate thermal bremsstrahlung where the e^- have a Maxwell-Boltzmann dist'n of speeds.

The probability dP that a particle has a velocity in the velocity range $d^3\vec{v}$ is

$$dP \propto e^{-E/kT} d^3\vec{v} = e^{-mv^2/2kT} d^3\vec{v}$$

For an isotropic dist'n of velocities, $d^3\vec{v} = 4\pi v^2 dv$ and

$$dP \propto v^2 e^{-mv^2/kT} dv$$

The thermal velocity averaged emissivity

$$\frac{dW(T,\omega)}{dV\,dt\,d\omega} = \frac{\int_{v_{min}}^{\infty} \frac{dW(v,\omega)}{d\omega\, dV\, dt} v^2 e^{-mv^2/2kT} dv}{\int_0^{\infty} v^2 e^{-mv^2/2kT} dv}$$

$d\nu\, dt\, d\omega$ $\quad \int_0 v^2 e^{-mv^2/2kT}\, dv$

where $v_{min} = \left(\frac{2h\nu}{m}\right)^{1/2}$ since $h\nu \leq \frac{1}{2}mv^2$ $(\omega = 2\pi\nu)$

Doing the integrals (try them!) & using $d\omega = 2\pi d\nu$ one obtains

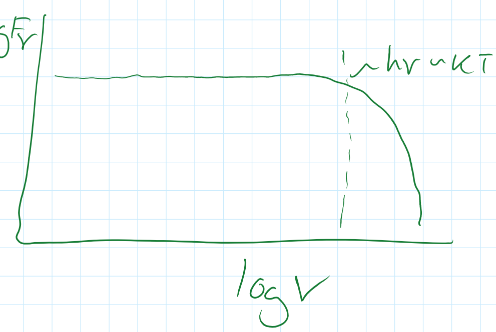
$$\frac{dW}{dV\,dt\,d\nu} = \frac{2^5 \pi e^6}{3mc^3}\left(\frac{2\pi}{3Km}\right)^{1/2} T^{-1/2} Z^2 n_e n_i e^{-h\nu/kT}\, \bar{g}_{ff}$$

Plug in the numbers: the ff emissivity ($erg\ s^{-1} cm^{-3} Hz^{-1}$)

$$\epsilon_\nu^{ff} \equiv \frac{dW}{dt\,dV\,d\nu} = 6.8 \times 10^{-38} Z^2 n_e n_i T^{-1/2} e^{-h\nu/kT}\, \bar{g}_{ff}$$

$\bar{g}_{ff}(T,\nu)$ is a velocity-averaged Gaunt factor. For most astrophysical plasmas $5 \gtrsim \bar{g}_{ff} \gtrsim 1$ & $\bar{g}_{ff} = 1$ is fine for most practical purposes.

The emission spectrum is flat



But this does not include the effects of absorption.

Numerical Values of gaunt factor

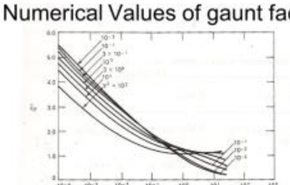


Figure 5.3 Numerical values of the gaunt factor $g_{ff}(\nu, T)$. Here the frequency variable is $u = 4.8 \times 10^{11}\nu/T$ and the temperature variable is $\gamma^2 = 1.58 \times 10^5 Z^2/T$. (Taken from Karzas, W. and Latter, R. 1961, Astrophys. J. Suppl., 6, 167.)

The total power produced by a ff plasma. Integrate ϵ_ν^{ff} over all freq.

$$\frac{dW}{dt\,dV} = \left(\frac{2\pi kT}{3m}\right)^{1/2} \frac{2^5 e^6 \pi}{3hmc^3} Z^2 n_e n_i \bar{g}_B$$

or

$$\epsilon_{ff} = 1.4\times 10^{-27} T^{1/2} n_e n_i Z^2 \bar{g}_B \quad (\text{erg s}^{-1}\text{ cm}^{-3})$$

$\bar{g}_B(T)$ varies from $1.1 \rightarrow 1.5$

- a value of 1.2 will typically get you within $\simeq 20\%$

Thermal Bremsstrahlung Absorption

An e^- moving in the Coulomb field of an ion can also absorb photons. We can relate this absorption process to the emission one. The thermal absorption most applicable in astrophysics.

Then we can use Kirchoff's Law: $j_\nu^{ff} = \alpha_\nu^{ff} B_\nu(T)$

$$4\pi j_\nu^{ff} = \frac{dW}{dt\,dV\,d\nu} \quad ; \quad \alpha_\nu^{ff} \text{ is the free-free absorption coefficient}$$

and $B_\nu(T)$ is the Planck or BB function

$$B_\nu(T) = \frac{2h\nu^3/c^2}{e^{h\nu/kT}-1} \quad (\text{erg cm}^{-2}\text{ s}^{-1}\text{ Hz}^{-1}\text{ ster}^{-1})$$

$\therefore$ From this and our previous expression

$$\alpha_\nu^{ff} = \frac{j_\nu^{ff}}{B_\nu(T)} = \frac{4e^6}{3mhc}\left(\frac{2\pi}{3km}\right)^{1/2} T^{-1/2} Z^2 n_e n_i \nu^{-3}\left(1-e^{-h\nu/kT}\right)\bar{g}_{ff}$$

or $\alpha_\nu^{ff} = (3.7\times10^{8})\, T^{-1/2} Z^2 n_e n_i \nu^{-3} \left(1 - e^{-h\nu/kT}\right) \bar{g}_{ff}$ (cm^{-1})