$$\alpha_\nu^{ff} = (3.7\times10^8)\, T^{-1/2} Z^2 n_e n_i \nu^{-3} \left(1 - e^{-h\nu/kT}\right) \bar{g}_{ff}$$

For $h\nu \gg kT$, $e^{-h\nu/kT} \to 0$; $\alpha_\nu \propto \nu^{-3}$

$h\nu \ll kT$, $e^{-h\nu/kT} \simeq 1 - \frac{h\nu}{kT}$

$$\to \frac{h\nu}{kT} \simeq 1 - e^{-h\nu/kT}$$

and $\alpha_{ff} = \frac{4e^6}{3mkc}\left(\frac{2\pi}{3km}\right)^{1/2} T^{-3/2} Z^2 n_e n_i \nu^{-2} \bar{g}_{ff}$

or $\alpha_\nu^{ff} = 0.018\, T^{-3/2} Z^2 n_e n_i \nu^{-2} \bar{g}_{ff}$ $\to$ imp. @ radio frequencies

A very useful expression for the optical depth

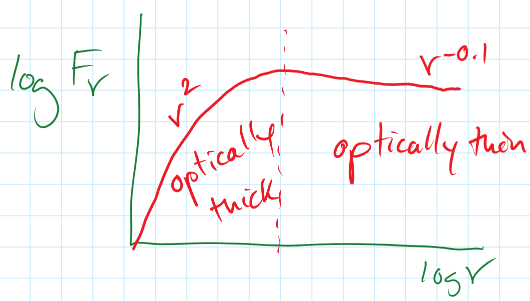
$\tau_\nu = \int \alpha_\nu ds$ may be obtained from a power-law fit

to the (ν, T)-dependence of the Gaunt factor

$$\tau_\nu = (8.235\times10^{-2})\, T^{-1.35} \left(\frac{\nu}{\mathrm{GHz}}\right)^{-2.1} E_M\, a(\nu,T)$$

$a(\nu,T)$ $\to$ order 1

where $E_M = Z^2 \int_0^s n_e n_i ds'$ is the emission measure in pc cm^{-6}



Synchrotron Radiation

Radiation from particles accelerated by $\vec{B}$-field. If $v \ll c$ then called cyclotron radiation & freq. of emission = freq. of gyration. If $v \sim c$, the spectrum can extend much higher than the gyration freq. This is called synchrotron radiation.

Total Emitted Power

E.o.M of particle in a B-field

$$\frac{d}{dt}(\gamma m \vec{v}) = \cancel{q\vec{E}}^{\,0} + \frac{q}{c}\vec{v}\times\vec{B}$$

Since the acc'n is always $\perp$ to $\vec{v}$, $|\vec{v}| = \text{constant}$

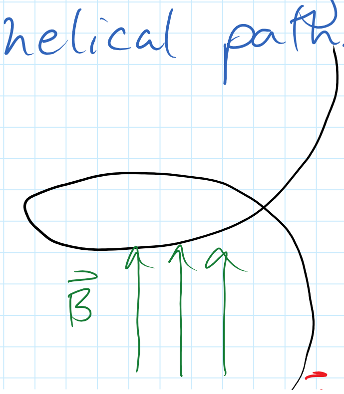
$$\therefore\ m\gamma \frac{d\vec{v}}{dt} = \frac{q}{c}\vec{v}\times\vec{B}$$

Seperate into components along the field, $\vec{v}_{\parallel}$, & normal to the field $\vec{v}_{\perp}$

$$\frac{d\vec{v}_{\parallel}}{dt} = 0 \quad ; \quad \frac{d\vec{v}_{\perp}}{dt} = \left(\frac{q}{\gamma m c}\right)\vec{v}_{\perp}\times\vec{B}$$

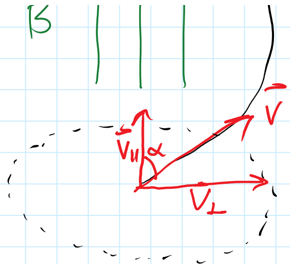
$\therefore\ \vec{v}_{\parallel} = \text{const.}$, but know $|\vec{v}| = \text{const.}$, so $|\vec{v}_{\perp}| = \text{const.}$

$\Rightarrow$ Uniform circular motion on the normal plane & uniform motion along the field. The particle traces a helical path.



$$\frac{dv_{\perp}}{dt} = \frac{q v_{\perp} B}{\gamma m c} = \frac{v_{\perp}^2}{r}$$

$$\therefore\ r = \gamma m c v$$



$$\therefore\ r = \frac{\gamma m c v_\perp}{qB}$$

$$\omega = 2\pi f = \frac{qB}{\gamma m c}$$

$$\therefore\ \frac{dv_\perp}{dt} = \omega v_\perp$$

From earlier, the total emitted power is

$$P = \frac{2q^2}{3c^3}\gamma^4\left(a_\perp^2 + \cancel{\gamma^2 a_\parallel^2}\right)^0 = \frac{2q^4}{3m^2c^3}B^2\beta_\perp^2\gamma^2 \quad \left(\beta_\perp = \frac{v_\perp}{c}\right)$$

Suppose we had an isotropic vel. dist'n or a random field orientation. Then the avg. power per e^- is found by averaging this formula over all angles for a given speed β

Let α = pitch angle, angle b/w $\vec{v}$ & $\vec{B}$

$$\therefore\ \langle \beta_\perp^2 \rangle = \frac{\beta^2}{4\pi}\int \sin^2\alpha \, d\Omega = \frac{2\pi\beta^2}{4\pi}\int_0^\pi \sin^2\alpha \, d\alpha = \frac{2\beta^2}{3}$$

$$\therefore\ \langle P \rangle = \frac{4}{3} \times \underbrace{\frac{8\pi q^4}{3m^2c^4}}_{\text{Thomson cross-section } \sigma_T} \times c \times \beta^2 \times \underbrace{\frac{B^2}{8\pi}}_{U_B \text{ mag. energy density}} \times \gamma^2$$

$$\therefore\ \langle P \rangle = \frac{4}{3}\sigma_T c \beta^2 \gamma^2 U_B$$

(or, when, $v \sim c$, $\langle P \rangle = \frac{4}{3}\sigma_T c \gamma^2 U_B$)