Monday, February 1, 2016

We had $\langle P \rangle = \frac{4}{3}\sigma_T c \beta^2 \gamma^2 U_B$

This expression leads to an interesting physical interp. As seen by the e^-, the magnetic energy flux flowing past is with magnitude cU_B.

So, multiply this flux (mag. energy/area/time) by scattering x-section σ_T (cm^2) and apply a boost factor γ^2 → get the synchrotron power. That is, synchotron rad'n can be viewed as the scattering of magnetic energy by relativistic electrons.

The radiating electron will lose energy at a rate

$$\frac{d\overset{\swarrow \text{energy}}{E}}{dt} = -P = -\frac{2q^4}{3m^2c^3} B^2 \beta_\perp^2 \gamma^2 = -\frac{2q^4}{3m^2c^3} B^2 \beta_\perp^2 \frac{\gamma^2 m^2 c^4}{m^2c^4}$$

$$= -\frac{2q^4}{3m^4c^7} B^2 \beta_\perp^2 E^2 = -AE^2$$

$$\text{where } A = \frac{2q^4 B_\perp^2}{3m^4c^7} \quad ; \quad \text{take } \beta \simeq 1$$

Solving: $-\frac{1}{A}\int_{E_0}^{E} \frac{dE}{E^2} = \int_0^t dt$

$$\frac{1}{A}\left(\frac{1}{E}\right)_{E_0}^{E} = t$$

$$\frac{1}{A}\left(\frac{1}{E} - \frac{1}{E_0}\right) = t$$

$$\frac{1}{E} - \frac{1}{E_0} = At$$

$$\frac{1}{E} = At + \frac{1}{E_0}$$

$$\frac{1}{E} = \frac{AtE_0 + 1}{E_0}$$

$A(E \quad E_0)$

$$\therefore \boxed{E = \frac{E_0}{1 + AE_0 t}}$$

or as $t \to \infty$, $E \propto \frac{1}{At}$

The half-life of the synchrotron e- is when $E = \frac{E_0}{2}$

$$\frac{E_0}{2} = \frac{E_0}{1 + AE_0 t_{1/2}}$$

$$\to t_{1/2} = \frac{1}{AE_0} = \frac{5.1 \times 10^8 s}{B_\perp^2}\left(\frac{mc^2}{E_0}\right) = \frac{5.1 \times 10^8 s}{\gamma B_\perp^2}$$

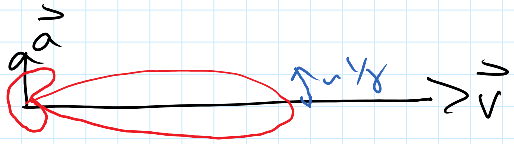
Q: How does one reconcile the decrease of γ here w/ the result of const. γ from the e.o.m?

- radiation reaction force

This gives a high energy cutoff in the spectrum → estimate an age of source/acc'n region.

Spectrum of Synchrotron Radiation (not-too-rigorous version)

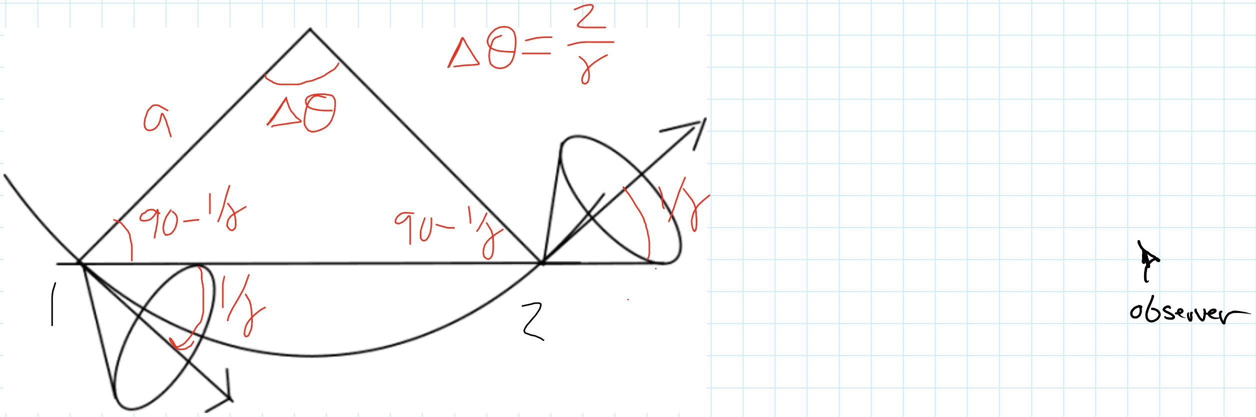
The radiation pattern emitted by the spiralling e- will be beamed in the forward direction



Thus, the observer will see a pulse of radiation confined to a time interval much smaller than the gyration period.

→ Spectrum will be spread over a freq. range much broader than $\frac{\omega_B}{2\pi}$

To get an idea of the size & spread of the freq. in the spectrum consider the emission of the particle at 2 points along its path



The observer will see radiation b/w these two points.

The distance Δs along the path, $\Delta s = a\Delta\theta$ where a = radius of curvature

but $\Delta\theta = \frac{2}{\gamma}$ so $\Delta s = \frac{2a}{\gamma}$

From e.o.m. $\gamma m \frac{\Delta \vec{v}}{\Delta t} = \frac{q}{c}\vec{v}\times\vec{B}$

$$|\Delta\vec{v}| = v\Delta\theta \quad ; \quad \Delta s = v\Delta t$$

$$\frac{|\Delta\vec{v}|}{\Delta t} = \frac{v^2\Delta\theta}{\Delta s} = \frac{q\cancel{v}B\sin\alpha}{c\gamma m}$$ ← pitch angle

$$\therefore \frac{\Delta\theta}{\Delta s} = q\frac{B\sin\alpha}{\gamma} = \omega_B \sin\alpha$$

$$\therefore \frac{\Delta\theta}{\Delta s} = \frac{qB\sin\alpha}{\gamma m c v} = \frac{\omega_B \sin\alpha}{v}$$

$$\therefore \; a = \frac{v}{\omega_B \sin\alpha}$$

$$\therefore \Delta s \simeq \frac{2v}{\gamma \omega_B \sin\alpha}$$

The particle is at point 1 at time t_1 ; at point t at time t_2 so $\Delta s = v(t_2 - t_1) \Rightarrow t_2 - t_1 \simeq \frac{\Delta s}{v} = \frac{2}{\gamma \omega_B \sin\alpha}$