Let t_1^A & t_2^A be the arrival times of radiation at the observer from points 1 & 2; $\Delta t^A = t_2^A - t_1^A$ is less than $t_2 - t_1$ by Doppler compression

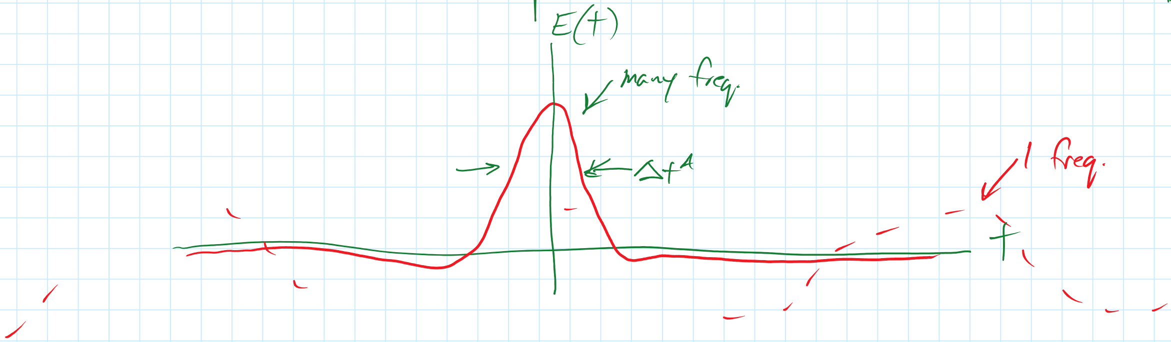
$$\therefore \Delta t^A = \frac{2}{\gamma \omega_B \sin\alpha}\left(1 - \frac{v}{c}\right)$$

In this situation, $\gamma >> 1$, so $\frac{1}{\gamma^2} = \left(1 - \frac{v^2}{c^2}\right) = \left(1 - \frac{v}{c}\right)\left(1 + \frac{v}{c}\right)$
$\simeq 2\left(1 - \frac{v}{c}\right)$

or $\left(1 - \frac{v}{c}\right) \simeq \frac{1}{2\gamma^2}$

$$\therefore \Delta t^A \simeq \frac{1}{\gamma^3 \omega_B \sin\alpha}$$

The width of the pulse is smaller than $\frac{1}{\omega_B}$ by γ^3!



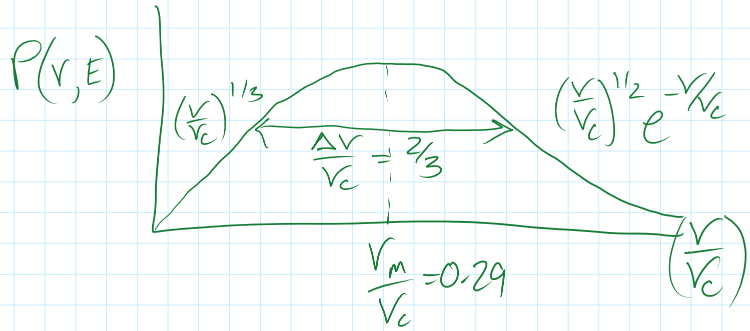
A Fourier transform of this pulse will be fairly broad b/c this pulse consists of many high freq. components

$$\text{bandwidth} = \Delta\omega \sim \Delta t^{-1} = \gamma^3 \omega_B \sin\alpha$$

Define a critical freq. $\omega_c = \frac{3}{2}\gamma^3 \omega_B \sin\alpha \propto \gamma^2$ since $\omega_B \propto \frac{1}{\gamma}$

$$\nu_c = \frac{3}{4\pi}\gamma^3 \omega_B \sin\alpha$$

The full calc. gives a spectrum for one particle:



The spectrum peak occurs at $\nu_m = 0.29\,\nu_c = 1.26\times10^6 B\gamma^2$ (Hz)

For $B \sim 3\,\mu G$ & $\gamma \simeq 10^3$, ν_c & $\nu_m \simeq 3\times10^7$ Hz (in the radio spect.)

If $\gamma \sim 1$, then $\nu \sim 30$ Hz, so rel. effects boosted ν by 10^6!

Volume Emissivity of a Power-Law Dist'n of e^-

Particle acc'n processes generally result in a power-law dist'n of particle energies

$$N(E)\,dE = K E^{-p}\,dE$$

where p is the energy spectral index (~ 2.5). To compute the emissivity j_ν we will assume (a) an uniform B-field & (b) isotropic dist'n

$$\therefore \quad j_\nu = \frac{1}{4\pi}\int_0^\infty \underbrace{P(\nu,E)}_{\text{spectrum of } e^- \text{ @ energy } E} N(E)\,dE$$

Approximate integral by assuming all e^- radiate at their ν_c

Then,
$$j_\nu\,d\nu = \frac{N(E)\,dE}{4\pi} \times P(E) = \frac{K E^{-p}}{4\pi} \times \frac{2q^4 B_\perp^2}{3m^2c^3}\left(\frac{E}{mc^2}\right)^2 dE$$

But $\nu = \nu_c = \frac{3}{\ldots}\,\omega_B \sin\alpha\,\gamma^3 = \frac{3}{\ldots}\left(\frac{eB_\perp}{\ldots}\right)\left(\frac{E}{\ldots}\right)^3$

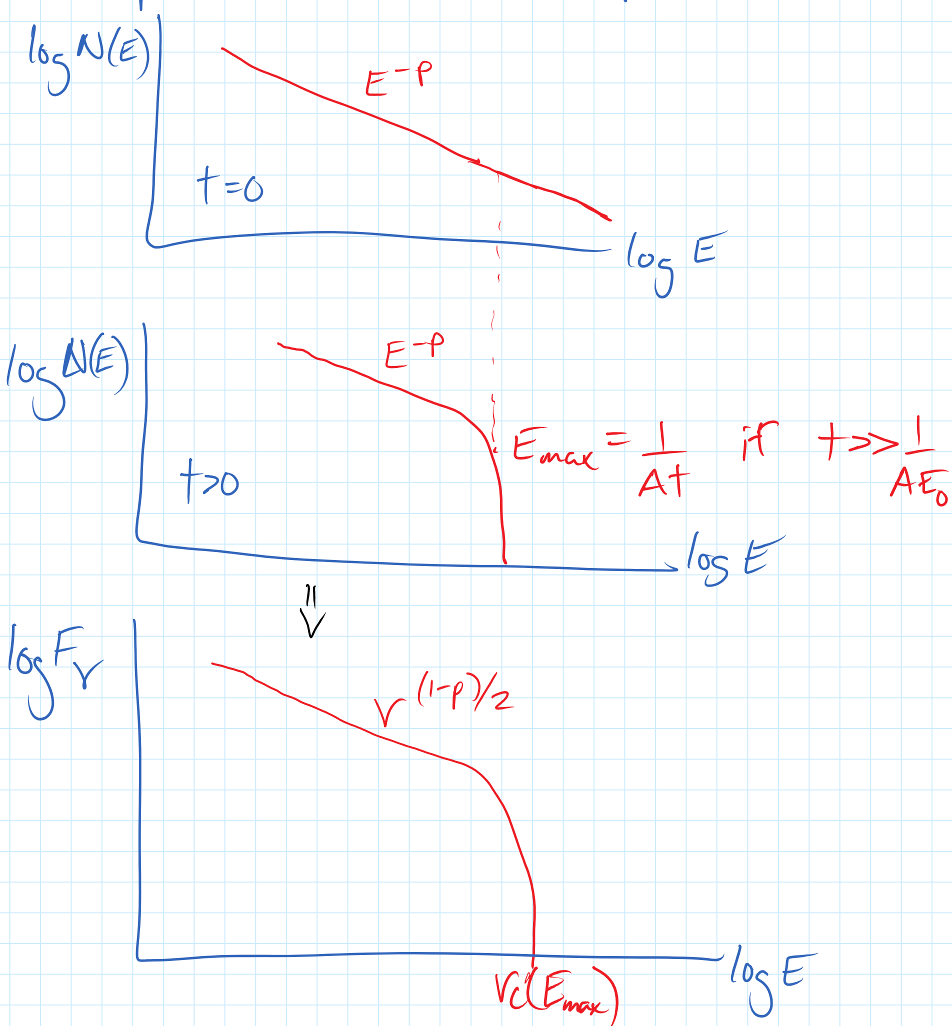
But $\nu = \nu_c = \frac{3}{4\pi}\,\omega_B \sin\alpha\, \gamma^3 = \frac{3}{4\pi}\left(\frac{eB_\perp}{\gamma mc}\right)\left(\frac{E}{mc^2}\right)^3$

Solve for E ; then calculate $dE = \cdots d\nu$, substitute and ASA

$$j_\nu \simeq \frac{1}{9}\left(\frac{q^3}{mc^2}\right)\left(\frac{3q}{4\pi m^3 c^5}\right)^{(p-1)/2} K B_\perp^{(p+1)/2}\, \nu^{(1-p)/2}$$

Formula is an approx. (see refs for exact formula) but has the correct dependeces on $B_\perp$; ν. That is, if $p = 2.5$ then $\nu^{-0.75}$

Note that radiation losses by high-energy particles will lead to an abrupt cutoff in the spectrum



Historically, non-thermal synchrotron is more important than thermal synchrotron b/c $P \propto \gamma^2$ while thermal energy is more evenly distributed, so unless the thermal plasma $T >> \frac{mc^2}{k}$, thermal synchrotron sources are too faint.

But, the SMBH @ Sgr A* has a spectrum that requires thermal synchrotron, so it can be relevent.