High-Energy Astrophysics

Problem Set 1

Due: 3/7/18

1. (a) Show that the transformation of acceleration is

$$
\begin{aligned}
a_x &= \frac{a'_x}{\gamma^3\sigma^3}, \\
a_y &= \frac{a'_y}{\gamma^2\sigma^2} - \frac{u'_y v}{c^2}\frac{a'_x}{\gamma^2\sigma^3}, \\
a_z &= \frac{a'_z}{\gamma^2\sigma^2} - \frac{u'_z v}{c^2}\frac{a'_x}{\gamma^2\sigma^3},
\end{aligned}
$$

where

$$\sigma \equiv 1 + \frac{v u'_x}{c^2}.$$

(b) If K' is the instantaneous rest frame of the particle, show that

$$
\begin{aligned}
a'_{\parallel} &= \gamma^3 a_{\parallel}, \\
a'_{\perp} &= \gamma^2 a_{\perp},
\end{aligned}
$$

where $a_{\parallel}$ and $a_{\perp}$ are the components parallel and perpendicular to the direction of v, respectively.

2. Suppose X-rays are received from a source of known distance L with a flux F (erg cm^{-2} s^{-1}). The X-ray spectrum has the form of Fig. 1. It is conjectured that these X-rays are due to bremsstrahlung from an optically thin, hot, plasma cloud, which is in hydrostatic equilibrium around a central mass M. Assume that the cloud thickness ΔR is roughly its radius R, $\Delta R \sim R$. Find R and the density of the cloud, ρ, in terms of the known observations and conjectured mass M (Hint: the virial theorem will be helpful). If $F = 10^{-8}$ erg cm^{-2} s^{-1}, $L = 10$ kpc, what are the constraints on M such that the source would indeed be effectively optically thin at 1 keV (for self-consistency)? Does electron scattering play any role?

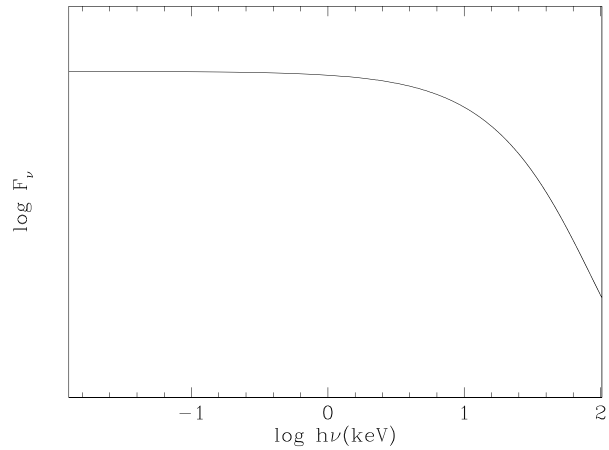

Figure 1:

3. (a) Use the basic equation from classical radiation theory,

$$\vec{E} \propto \hat{n} \times (\hat{n} - \vec{\beta}) \times \dot{\vec{\beta}},$$

to demonstrate that $\vec{E}$ lies along $\dot{\vec{\beta}}$ for synchrotron radiation from ultra-relativistic electrons. Assume that the observer lies at the pulse center; i.e., $\hat{n}$ and β are parallel at the peak of the pulse. Explain how polarization observations of synchrotron sources measure the direction of $\vec{B}$.

(b) Do the same for (i) non-relativistic electrons ($\beta \ll 1$) and (ii) partially relativistic electrons ($\beta \sim 1/2$), all radiating in a magnetic field. For these cases, assume that $\hat{n}$ is parallel to the field $\vec{B}$ and that the pitch angle is 90 degrees. No need to re-describe the interpretation of polarization observations.

(c) In class, we used the relativistic form of the Larmor formula to derive the total power emitted by an ultra-relativistic ($\gamma \gg 1$) synchrotron electron. We found that $P \propto \gamma^2 \propto E^2$, where E is the electron kinetic energy. Use the same approach to find the corresponding relation for an arbitrary Lorentz factor γ. Express the result in two forms, one where γ is the only independent variable describing the electron's energy (aside from the pitch angle θ, of course) and another where v/c is the only independent variable. In the second case, take the limit $v \ll c$ and show that $P \propto E^m$. What is the value of m? In particular, does $m = 2$ as in the ultra-relativistic limit?

4. It can be shown that the synchrotron absorption coefficient for an isotropic electron distribution $N(E)$ is

$$\alpha_\nu = -\frac{c^2}{8\pi\nu^2} \int_0^\infty P(\nu, E) E^2 \frac{d}{dE} \left[\frac{N(E)}{E^2} \right] dE,$$

where $P(\nu, E)$ is the spectrum for an individual electron. In class, we obtained a good approximation for the synchrotron emissivity j_ν due to a power-law distribution of electron energies by assuming that all the electrons radiate only at their critical frequencies ν_c rather than over a broad spectrum. The broad-band behavior turns out to be mostly buried in the convolution with the electron energy spectrum. Use the same approach to find an expression for α_ν for a power-law distribution of electron energies.

5. In class, we saw that low energy photons are *on average* boosted by a factor γ^2 in energy by scattering off relativistic electrons with a characteristic γ. Indeed, the total scattered power $P_{\rm IC}$ is proportional to $\gamma^2 U_R$ for "soft" radiation ($\gamma\epsilon \ll mc^2$). But, for specific incident angles θ and specific scattering angles θ'_S in the rest frame, the boost factor can be of order unity or even smaller in some instances. Consider the following three special cases and evaluate the factor relating ϵ_1 and ϵ. Assume that the Lorentz factor is large enough so that $(1 - v/c) = 1/(2\gamma^2)$ and $(1 + v/c) = 2$.
   (a) $\theta = 0$, $\theta_s = \theta'_s = \theta_a$
   (b) $\theta = 0$, $\theta_s = \theta'_s = \pi$
   (c) $\theta = \theta_a$, $\theta_s = \theta'_s = \pi$
   Here, $\theta_a = \langle\theta\rangle = \pi/2 \rightarrow \langle\cos\theta\rangle = \cos\theta_a = 0$.

6. Consider the observed X-ray source of Problem 2. From the deduced characteristics of the source, determine a lower limit to the central mass M such that inverse Compton effects in the emission mechanism are negligible.

7. Show that the photon energy in the electron rest frame is small compared to mc^2 for the following two cases:

   i. Electrons with $\gamma \sim 10^4$ scattering synchrotron photons produced in a magnetic field $B \sim 0.1$ G (typical of compact radio sources).

   ii. Electrons with $\gamma \sim 10^4$ scattering the 3 K photons of the cosmic microwave background.