High-Energy Astrophysics
Problem Set 2 — Solutions

1. (a) If each photon has a typical energy of 1 keV, then 220 photons$\approx 3.5 \times 10^{-7}$ erg. The observed flux is then $F = 1.5 \times 10^{-14}$ erg cm^{-2} s^{-1}.

   (b) Assuming isotropic emission, $L = 4\pi d^2 F = 1.1 \times 10^{32}$ erg s^{-1}.

   (c) From the class notes, the Bondi accretion rate is

$$\dot{M} = \pi G^2 M^2 \frac{\rho(\infty)}{c_s(\infty)} \left[\frac{2}{5-3\gamma}\right]^{5-3\gamma/2(\gamma-1)},$$

   where the γ-dependent factor is ≈ 1 when $\gamma = 5/3$. Assuming mostly hydrogen gas, $\rho = 4.3 \times 10^{-23}$ g cm^{-3}, and

$$c_s^2 \approx \frac{P}{\rho} \approx \frac{nkT}{\rho}$$

   Thus, $c_s \approx 3.5 \times 10^7$ cm s^{-1} and

$$\dot{M} = 8.7 \times 10^{20} \text{ g s}^{-1} = 1.4 \times 10^{-5} \text{ M}_\odot \text{ yr}^{-1}.$$

   (d) $\eta = \frac{L}{\dot{M}c^2} = 1.4 \times 10^{-10}$

   (e) $L_{\rm Edd} = (1.3 \times 10^{38}) \left(\frac{M}{M_\odot}\right) = 5.2 \times 10^{44}$ erg s^{-1}.

   $\frac{L}{L_{\rm Edd}} = 2.2 \times 10^{-13}$

   (f) The uncertainties that go into the Eddington ratio are the distance to Sgr A*, the mass of the BH, and the measurement of L. Most of these are fairly well understood. The radiative efficiency estimate is more uncertain because, in addition to the uncertainties in the distance and isotropy of the emission, it is assuming that the accretion rate into the BH is the same as the Bondi rate. The Bondi rate is likely only to be an upper limit.

2. Following the procedure described in class and using $\kappa_{\rm es} = 0.4$ cm^2 g^{-1} gives

$$\begin{aligned}
\Sigma &= (9.7 \times 10^4)\alpha^{-4/5} \left(\frac{\eta}{0.1}\right)^{-3/5} \left(\frac{\dot{M}}{\dot{M}_{\rm Edd}}\right)^{3/5} \left(\frac{R}{R_{\rm Sch}}\right)^{-3/5} \left(\frac{M}{M_\odot}\right)^{1/5} f^{12/5} \\
H &= (4.6 \times 10^3)\alpha^{-1/10} \left(\frac{\eta}{0.1}\right)^{-1/5} \left(\frac{M}{M_\odot}\right)^{9/10} \left(\frac{\dot{M}}{\dot{M}_{\rm Edd}}\right)^{1/5} \left(\frac{R}{R_{\rm Sch}}\right)^{21/20} f^{4/5}
\end{aligned}$$

$$
\begin{aligned}
\rho &= (21)\alpha^{-7/10}\left(\frac{M}{M_\odot}\right)^{-7/10}\left(\frac{\eta}{0.1}\right)^{-2/5}\left(\frac{\dot{M}}{\dot{M}_{\rm Edd}}\right)^{2/5}\left(\frac{R}{R_{\rm Sch}}\right)^{-33/20} f^{8/5} \\
T_c &= (8.2\times 10^{8})\alpha^{-1/5}\left(\frac{M}{M_\odot}\right)^{-1/5}\left(\frac{\eta}{0.1}\right)^{-2/5}\left(\frac{\dot{M}}{\dot{M}_{\rm Edd}}\right)^{2/5}\left(\frac{R}{R_{\rm Sch}}\right)^{-9/10} f^{8/5} \\
\tau &= (3.9\times 10^{4})\alpha^{-4/5}\left(\frac{\eta}{0.1}\right)^{-3/5}\left(\frac{\dot{M}}{\dot{M}_{\rm Edd}}\right)^{3/5}\left(\frac{R}{R_{\rm Sch}}\right)^{-3/5}\left(\frac{M}{M_\odot}\right)^{1/5} f^{12/5} \\
\nu &= (1.5\times 10^{12})\alpha^{4/5}\left(\frac{M}{M_\odot}\right)^{4/5}\left(\frac{\eta}{0.1}\right)^{-2/5}\left(\frac{\dot{M}}{\dot{M}_{\rm Edd}}\right)^{2/5}\left(\frac{R}{R_{\rm Sch}}\right)^{3/5} f^{8/5} \\
v_r &= (7.6\times 10^{6})\alpha^{4/5}\left(\frac{\eta}{0.1}\right)^{-2/5}\left(\frac{\dot{M}}{\dot{M}_{\rm Edd}}\right)^{2/5}\left(\frac{M}{M_\odot}\right)^{-1/5}\left(\frac{R}{R_{\rm Sch}}\right)^{-2/5} f^{-12/5}
\end{aligned}
$$

Calculating H/R,

$$
\frac{H}{R} = (0.016)\alpha^{-1/10}\left(\frac{\eta}{0.1}\right)^{-1/5}\left(\frac{\dot{M}}{\dot{M}_{\rm Edd}}\right)^{1/5}\left(\frac{M}{M_\odot}\right)^{-1/10}\left(\frac{R}{R_{\rm Sch}}\right)^{1/20} f^{4/5},
$$

we see that the disk is thin, as required.

Note I followed Frank, King & Raine (2002) and used $\mu = 0.615$ for the mean molecular weight. This is appropriate for a fully ionized 'cosmic' mixture of gases.

3. The solutions are obtained by following the same procedure, but using radiation pressure ($P = (4\sigma/4c)T_c^4$) as the only source of pressure.

$$
\begin{aligned}
\Sigma &= (0.43)\alpha^{-1}\left(\frac{\eta}{0.1}\right)\left(\frac{\dot{M}}{\dot{M}_{\rm Edd}}\right)^{-1}\left(\frac{R}{R_{\rm Sch}}\right)^{3/2} f^{-4} \\
H &= (2.2\times 10^{6})\left(\frac{\eta}{0.1}\right)^{-1}\left(\frac{M}{M_\odot}\right)\left(\frac{\dot{M}}{\dot{M}_{\rm Edd}}\right) f^{4} \\
\rho &= (1.9\times 10^{-7})\alpha^{-1}\left(\frac{M}{M_\odot}\right)^{-1}\left(\frac{\eta}{0.1}\right)^{2}\left(\frac{\dot{M}}{\dot{M}_{\rm Edd}}\right)^{-2}\left(\frac{R}{R_{\rm Sch}}\right)^{3/2} f^{-8} \\
T_c &= (3.7\times 10^{7})\alpha^{-1/4}\left(\frac{M}{M_\odot}\right)^{-1/4}\left(\frac{R}{R_{\rm Sch}}\right)^{-3/8} \\
\tau &= (0.17)\alpha^{-1}\left(\frac{\eta}{0.1}\right)\left(\frac{\dot{M}}{\dot{M}_{\rm Edd}}\right)^{-1}\left(\frac{R}{R_{\rm Sch}}\right)^{3/2} f^{-4} \\
\nu &= (3.6\times 10^{17})\alpha\left(\frac{M}{M_\odot}\right)\left(\frac{\eta}{0.1}\right)^{-2}\left(\frac{\dot{M}}{\dot{M}_{\rm Edd}}\right)^{2}\left(\frac{R}{R_{\rm Sch}}\right)^{-3/2} f^{8}
\end{aligned}
$$

$$v_r = (1.8 \times 10^{12})\alpha \left(\frac{\eta}{0.1}\right)^{-2} \left(\frac{\dot{M}}{\dot{M}_{\rm Edd}}\right)^{2} \left(\frac{R}{R_{\rm Sch}}\right)^{-5/2} f^4$$

To calculate the dividing line in radius between the gas pressure and radiation pressure dominated regions, first calculate the expressions for gas pressure and radiation pressure using the above radiation pressure dominated solutions:

$$P_{gas} = \frac{\rho k T_c}{\mu m_p} = (9.4 \times 10^8)\alpha^{-5/4} \left(\frac{M}{M_\odot}\right)^{-5/4} \left(\frac{\eta}{0.1}\right)^{2} \left(\frac{\dot{M}}{\dot{M}_{\rm Edd}}\right)^{-2} \left(\frac{R}{R_{\rm Sch}}\right)^{9/8} f^{-8}$$

$$P_{rad} = \frac{4\sigma}{3c} T_c^4 = (4.7 \times 10^{15})\alpha^{-1} \left(\frac{M}{M_\odot}\right)^{-1} \left(\frac{R}{R_{\rm Sch}}\right)^{-3/2}$$

Then set the two pressures equal and solve for radius:

$$\left(\frac{R}{R_{\rm Sch}}\right) f^{-64/21} = (356)\alpha^{2/21} \left(\frac{M}{M_\odot}\right)^{2/21} \left(\frac{\eta}{0.1}\right)^{-16/21} \left(\frac{\dot{M}}{\dot{M}_{\rm Edd}}\right)^{16/21}$$